Department of Mathematics

Math 4009 – Differential Equations
Semester II, 2023-24

Time: TBA
Room: TBA
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Course page: https://online.uprrp.edu/course/view.php?id=72602

Course Description

MATH 4009.Differential Equations. *Three credits. Three hours of lecture per week. Prerequisite: MATH 3152. Recommended MATH 4031 Linear Algebra*

Ordinary differential equations of the first order. Linear differential equations with constant coefficients. Linear differential equations of the second order. Systems of differential equations. Applications.

Objectives

After this course the student is expected to:

1. to have developed their skills in thinking logically, formulating precise mathematical arguments, solving problems and presenting solutions in the context differential equations.
2. to have become familiar with the main definitions, concepts, results and methods of proof relating to each part of the syllabus.
3. be able to quote the definitions and results, and to reproduce the proofs of some key results.
4. be able to solve problems relating to the material covered and present the solutions in a good mathematical style. These might be straightforward applications of the definitions and results but might also be problems of a more testing nature.

Syllabus

1. **Overview of Differential Equations** [3 weeks]
   (a) The definition of an ordinary differential equation.
   (b) Examples of different types of differential equations.
   (c) Definition of a solution. Maximal solutions.
   (d) Direction fields and their properties.

(e) Differentiable equations soluble by elementary methods.

(f) Non dimensionalization.

(g) Existence and uniqueness of solutions.

2. **Review of linear algebra** [2 weeks]

   (a) Definition of a vector space and examples.

   (b) The subspace criterion.

   (c) Maximal linearly independent and minimal spanning sets.

   (d) Bases and the definition of dimension of a vector space.

   (e) Linear maps

   (f) The kernel of a linear map.

   (g) Linear equations.

3. **Linear differential equations of first order** [1 week]

   (a) Solution of homogeneous equation

   (b) Inhomogeneous case.

   (c) Formula for a particular solution.

   (d) Special cases of particular solutions for constant, exponential, sinusoidal, step function and delta function inputs.

   (e) Applications.

4. **Linear differential equations of second order** [4 weeks]

   (a) Dimension of the solution space of the homogeneous equation.

   (b) The Wronskian and linear independence.

   (c) The inhomogeneous second order linear differential equation.

      i. Method of undetermined coefficients.

      ii. Variation of parameters. Connection

   (d) Linear differential equations with constant coefficients.

   (e) Solution of the homogeneous equation where the characteristic polynomial has distinct real roots, repeated roots, complex roots.

   (f) Qualitative and quantitative analysis of the forced damped oscillator equation

      i. Cases of zero damping, small and heavy damping.

      ii. Resonance and beats. Amplitude modulation.

      iii. Solution techniques using complex numbers.

      iv. Solution by variation of parameters and derivation of the Greens function.

v. Convolution and impulse response.

5. **Autonomous differential equations in one dimension** [2 weeks]
   (a) The initial value problem $x' = v(x), \quad x(t_0) = x_0$. where v is continuous on an interval.
      i. Implicit form of the solution.
      ii. Examples of non uniqueness of solutions when $v(x_0) = 0$
      iii. Uniqueness at a critical point when v is differentiable.
   (b) Symmetry considerations.
      i. Time invariance
      ii. The cases when v is odd or even.
   (c) The phase space of a differential equations of first order.
      i. Phase line and phase space.
      ii. Stable equilibria, unstable equilibria.
6. **Exact differential equations, and conservative vector fields** [1 week]
   (a) Integrating factors.
   (b) Solution curves and trajectories of the associated first order system.
   (c) A necessary condition for exactness. Conditions for sufficiency.
   (d) Orthogonal trajectories.
7. **The Laplace Transform** [1 week]
   (a) Definition of the Laplace transform
   (b) Convergence.
   (c) Shift and other theorems.
   (d) Convolution.
   (e) The Laplace transform of a differential equation with non-constant coefficients.
8. **Solutions in Power Series** [1 week]
   (a) Convergence of series.
   (b) Remarks on the nature of functions and power series.
   (c) Solution in series of first and second order equations.
   (d) Singular points.
   (e) Frobenius series and method.
   (f) The indicial equation.
   (g) The Bessel equation.

Text

1. Walter Leighton, *A First Course in Ordinary Differential Equations*, Wadsworth 1976.

Bibliography

1. Arnold, V.I. *Ordinary Differential Equations* Cambridge, MA: MIT Press, 1973, 1978.
2. D.K.Arrowsmith and C.M.Place, *Ordinary Differential Equations*. Chapman and Hall 1982.
3. Birkhoff, Garrett and Rota, Gian-Carlo.*Ordinary Differential Equations*, New York, NY: John Wiley, 1969, 1989. Fourth Edition.
4. Boyce, William E. and DiPrima, Richard C. Elementary *Differential Equations and Boundary Value Problems* , New York, NY: John Wiley.
5. Bronson, Differential Equations (Schaum Outline), Mcgraw Hill 1973.
6. Coddington, Earl A. *An Introduction to Ordinary Differential Equations*, Mineola, NY: Dover, 1989.
7. Hirsch, Morris W. and Smale, Stephen. *Differential Equations, Dynamical Systems, and Linear Algebra* New York, NY: Academic Press, 1974.
8. Hubbard, John H. and West, Beverly H.*Differential Equations: A Dynamical Systems Approach*, New York, NY: Springer-Verlag, 1991.
9. Sanchez, David A. *Ordinary Differential Equations and Stability Theory: An Introduction* Mineola, NY: Dover, 1979.

Electronic Resources

1. Math 4009 Course Page
   (`http://pennance.us/mathematics-course-pages/`).
2. Online Lectures from MIT by Gilbert Strang.
   (`https://ocw.mit.edu/courses/res-18-009-learn-differential-equations-up-close-with-`
3. Wolfram Alpha, http://www.wolframalpha.com

Instructional Strategies and Policies

Instructional strategies: lectures, readings, exercises, exams and online videos.

Evaluation

Your grade will be based upon three partial exams three exams (80%) and homework (20 %).

The grading scale will be no worse than the following: A: $90-100$; B: $80-89$; C: $65-79$; D: 50-64; F: < 50.