

ON THE INSTRUCTION OF CALCULUS

by Silviu Teleman

One of the most important problems facing the American School today is the Instruction of Calculus. We deliberately use the word “instruction” and not “teaching” because teachers are now disappearing, being replaced by “instructors”.

The main problem with Calculus originates, in its difficulty; one has to find simpler forms of presenting its main ideas, which should be more accessible and which should satisfy better the needs of the students.

This desire –we could say a democratic desire, because it is required by the society at large– was understood by the Calculus Instructing Community, which consequently organized into panels, committees, commissions and consortiums, who were given the task to produce the “Calculus Reform”. It happened that the historical moment was as favorable as possible for the Calculus Instructing Community, due to the appearance of the High Speed Electronic Calculators. Had the task of “Reforming Calculus” fallen on the brows (shoulders) of the Calculus Instructors sixty years ago, they could not have produced a table of values such as the following:

t	$\frac{\sin t}{t}$
± 0.1000	0.99833
± 0.0100	0.99998
± 0.0010	1.00000
± 0.0001	1.00000

which leads immediately to the conclusion that

$$\lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$$

(cf. *Calculus*, Harvard Calculus Consortium textbook, p. 251).

The use of computers, and calculators, in teaching (i.e., instructing) Calculus created an enormous advantage for the Calculus Instructing Community: they do not need to know Calculus any more. It is enough to buy a complete library of Calculus Instructing Disks (like Derive, Mathematica, etc.), and the instructor has no other task but to teach (instruct) the student how to push the buttons. The answer appears on the screen. What counts is to obtain a correct answer, whatever be the method. Understanding is not necessary any more.

It is true that all fundamental theorems of Calculus hold only when stated about the real number system, and they fail if we use only the rational rational numbers: the “Intermediate Value Theorem” the Rolle Theorem, the “Mean Value Theorem”, the “First Fundamental Theorem of Calculus”, etc., unfortunately, do not live in $\mathbb{Q}$; and, what is yet more unfortunate, irrational numbers are not swallowed by computers. What is the solution to this awful dilemma? There is a solution, to be carried out by steps.

1. Return to Babylonian Mathematics, by eliminating the irrational numbers; after all, these do not exist. Babylonians, as is very well known, by the Calculus Instructing Community, were very good applied mathematicians (unfortunately, the name of not even one of them is preserved -who cares?), astronomers and astrologers. They were also adept at solving (approximately, of course) cubic equations. Today we cannot avoid it –we have to be satisfied with the (approximate, of course) solutions given by computers, and we are happy with them. We do not need exact solutions any more.

The Calculus Instructing Community knows that, in solving a problem, a computer can become “wild”: the answer might be very far from the “real”, solution. This is no problem. First of all, it might be considered to be an “Act of God”, and so everybody will be released of any responsibility. On the other hand, it is well known that even great mathematicians, in the past, but also today, committed, and commit, gross errors; why should then computers not be allowed to commit errors?

2. There are some difficulties in performing step #1: namely the letter π (I do not say the “number π ”, since, this being transcendental, does not exist). However, even after the “Calculus Reform”, as exemplified by the HCC textbook, the letter π appears in some formulas. This can be remedied. First of all, π is a letter from the alphabet of a dead language: it has to be replaced. It will suit better the needs of the students to write the formula for the area of the disk of radius R as follows

$$(\text{pi})R^2;$$

after all, the pronunciation is the same, it uses only the symbols of a modern alphabet and it is pronounced exactly like

$$(\text{pie})R^2;$$

This innovation might help improve the understanding and memorization of the formula, because, usually, a pie has the shape of a disk (whether it is an apple pie or a pumpkin pie is immaterial).

3. Once we agree to develop Calculus only on the collection of the rational numbers (I do not say “set of the rational numbers” because this expression might be interpreted as requiring the Theory of Sets, which was introduced in Mathematics by such criminal Platonists as Cantor, and others), it is appropriate to point out that we do not need all of them: only those which can be fed into a computer; for example, a number like

$$10^{-10^{10}}$$

is useless, because the number of elementary particles in the Universe is less than 10^{100} ; of what practical use could be

$$10^{10^{10}}$$

or its inverse, after all?

4. Also, fractions have to be eliminated; they are too difficult. Use, instead, decimal fractions only, subject to the condition that they be not very long. A decimal fraction is a sequence of digits, from the collection

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

or, better, to use the custom of the Telephone Company,

1, 2, 3, 4, 5, 6, 7, 8, 9, 0

Examples:

- (a) 731654903
is an acceptable decimal fraction;
- (b) 547398036789363275902163866340942752733765439
is not, being too long.

Usually, a dot is present.

Examples:

- (a) .53875301
- (b) 371.78093268.

5. The instructor will show great care in instructing the students how to add two such numbers.

Example: $421.02902 + 53.4512$ is to be computed according to the following rule: put the numbers one over the other, with the dot over the dot

$$\begin{array}{r} 421.02902 \\ +53.4512 \end{array} \quad \text{or} \quad \begin{array}{r} 53.4512 \\ + 421.02902 \end{array}$$

(the order is immaterial). Draw a line under them

$$\begin{array}{r} 421.02902 \\ +53.4512 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 53.4512 \\ + 421.02902 \\ \hline \end{array}$$

and begin adding from the right to the left (this will be easier for the right-handed students; a little more difficult for left-handed students).

6. The problems assigned to the students should be of a reasonable difficulty. One should proceed by stages as follows.

Stage I. Calculus on the collection of integers, (again, the word “set” must be avoided, due to its non-existence status).

Examples:

Consider the function

x	$f(x)$
-1	2
0	-3
1	-1
2	3
3	4

Problem. Find the derivative at 2.

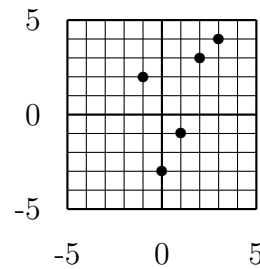
Answer:

$$f'(2) = \frac{4 - 3}{3 - 2} = \frac{1}{1} = 1$$

(sorry, for the use of fractions).

Problem. Is the function continuous at 0?

Answer: Answer: the graph of the function is



The function is not continuous at 0 because it jumps too much.

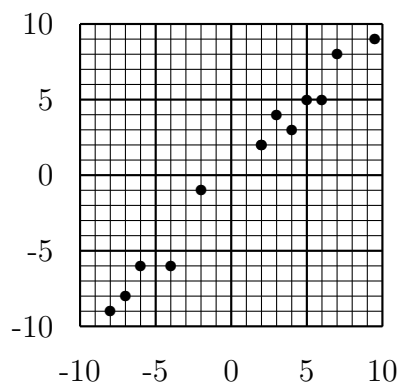
Problem. Is the function continuous at 2?

Answer: Yes, from the right, because it does not jump very much; it is discontinuous from the left, because it jumps too much.

Stage II. Calculus on the collection of numbers with one digit after the dot, like

$$2.3; \quad 4.5; \quad -1.2; \quad 10.3.$$

Here one should use scientific paper. Example: the graph of a function looks like this



Question: is the function linear? Hint: put one of your eyes on the paper and look along the graph; if you see only one point then the function is linear.

Stage III. Two digits after the dot, and so on, but not very far.

7. It looks as if Calculus can do without the notion of limit; it is true: the Babylonians did not have it and computers cannot compute limits. Using limits for the computation of derivatives on a calculator gives the ridiculous result that all derivatives are equal to 0. This proves, beyond any doubt, that limits have to be eliminated from Calculus. After all, when we write

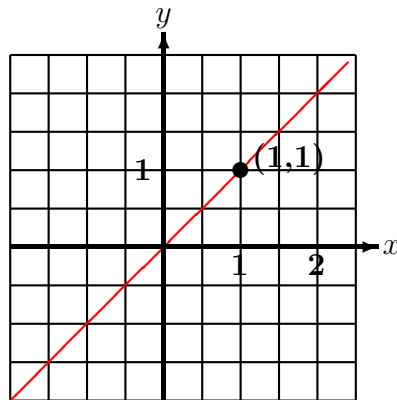
$$\lim_{x \rightarrow 0} x = 0$$

we have to eliminate the value $x = 0$ from consideration. But if 0 is eliminated, a number $x > 0$ can be made as small as we wish, without limit. Hence, the limit does not exist. The notion is self-contradictory and not even old-fashioned mathematicians admit contradictions.

8. There is a problem with the letter e (the number e does not exist, because it is transcendental). This letter is used in some formulas, by biologists and bankers, but only as a reminder, which sends them to the appropriate tables or data-banks, stored on disks, and, therefore, it does not pose a real problem.
9. The new method of instructing Calculus has a great advantage: since it tends to eliminate the criminal deductive method of exposition, à la Euclid, it could be further adapted to suit the needs of chimps (average I.Q. ≈ 7 , much greater than that of computers, which equals 0). I saw (on Discovery Channel, if I recall well) a (female; it seems that they are more intelligent than males) chimp, who was instructed to push buttons as an answer to some stimuli: she was shown images moving from left to right, or from right to left, and she had to push the corresponding button. She did it faster than I could have done it. I understand that one could instruct Calculus to chimps (at a more elementary level, for the beginning, of course), in order to solve problems like these:

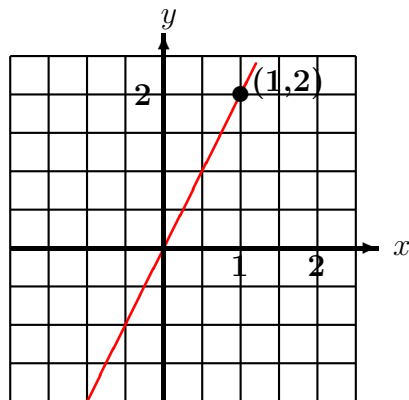
Situation problem #1: Find the slope

Answer: 1



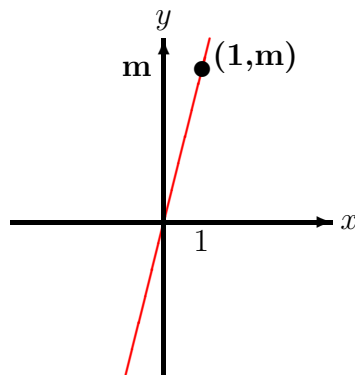
Situation problem #2: Find the slope

Answer: 2



Situation problem #m: Find the slope

Answer: m .



Hint: look at the point which stays over 1; if the second letter is m , the slope is m . Of course, it is better to speak about letters, because numbers are invisible. I think that for an intelligent chimp it would not be difficult, after some practice, of course, to push the appropriate button.

It goes without saying that Skinner's theory of reinforced stimuli is to be applied, by rewarding the chimp with a chocolate candy, whenever she succeeds. This idea is not original; it is prompted by the kindness shown by a speaker who, before giving a talk about mathematical pedagogy (or pedagogical mathematics, I do not recall which; it was something new, in any case) offered all attending persons a chocolate candy each, from a box which was opened in front of us, just to prove that the candies were not poisoned. Of course, the candy I ate tasted very good, and made the speech more palatable.

Long Live Nintendo Calculus!

Silviu Teleman – January 1994

Endnote

The following statements were made by university professors of mathematics, on various occasions, in implicit or explicit support of the "Calculus Reform". They deserve to be immortalized.

Irrational numbers do not exist.

I teach the derivative without the notion of limit.

I teach the integral without the notion of limit.

We should return to Babylonian Mathematics.

Bourbaki is a criminal.

In Mathematics we should be concerned with real-life problems.

Who tells you that the Universe is infinite ?

Traditional Mathematics became too difficult and, therefore, we must return to Approximate Mathematics.

It is true that computers cannot prove that $\sqrt{2}$ is irrational, –but this is not needed.

Mathematics is not based on Logic.

The enclosed essay is an attempt to prove the correctness of these statements and, implicitly, is meant to support the "Calculus Reform".

Silviu Teleman – February 18, 1994