Homework on Isomorphism of Simple Graphs

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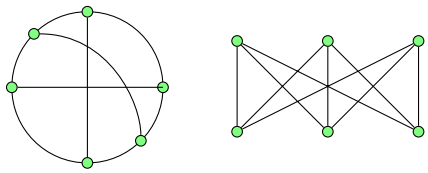
1. Decide if the following graphs are isomorphic. Prove your answer.



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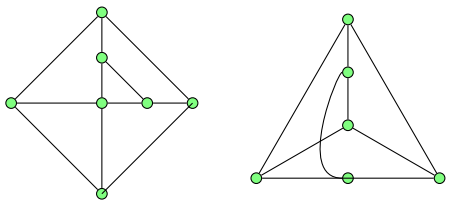

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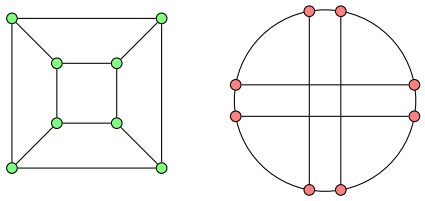

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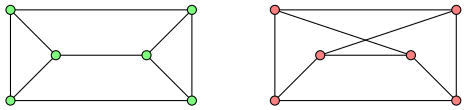

5. Let $g(V)$ the set of all simple graphs with vertex set V where $|V| = n$ and let $g_m(V) = \{G \in g(V) : |E(G) = m|\}$.

6. Let $\mathcal{G}_4$ be the set of all simple undirected graphs with vertex set $V = \{1, 2, 3, 4\}$. Let $\mathcal{G}_4(m)$ be the number of different graphs in $\mathcal{G}_4$ with m edges $0 \leq m \leq 6$. Find $|\mathcal{G}_4(m)|$ for each m.

7. Show that isomorphism of simple undirected graphs is an equivalence relation.

8. Let $\mathcal{G}_4$ be the set of all simple undirected graphs with vertex set $V = \{1, 2, 3, 4\}$.

   (a) The isomorphism relation partitions $\mathcal{G}_4$ into 11 equivalence classes. (i.e., up to isomorphism, there are 11 fundamentally different graphs on four vertices). Sketch a representative for each equivalence class.

   (b) For each class representative in (a) find the number of automorphisms.

   (c) For each class representative find the number of different graphs isomporhic to the representative. (i.e., calculate the class size).

   (d) For each class representative in (a) compute the product of the class size and the number of automorphisms. What do you observe?

9. Let $\mathcal{T}_n$ be the set of all spanning trees with vertex set $V = \{1, 2, \ldots, n\}$

   (a) How many non-isomorphic graphs are there in $\mathcal{T}_5$? (i.e., how many equivalence classes are there.). Sketch a representative of each class.

   (b) For each representative in (a) find the number of automorphisms.

   (c) Find the number of different graphs in each equivalence class.

   (d) Verify that the number of different spanning trees in $\mathcal{T}_5$ is 125.

[1] http://pennance.us