

**Math 3026–Elementary Combinatorics**  
**Prof. Philip Pennance <http://pennance.us/>**

In each of the following carefully prove your answer by reference to the appropriate combinatorial principle.

1. A multiple choice exam has 20 questions each with 5 possible answers. In how many ways can this exam be answered.?
2. (a) In how many ways can a committee of 2 persons be chosen from a set of 7 persons?  
(b) In how many ways can a president and a vicepresident be selected from a committee of 7?
3. Suppose there are 6 roads between San Juan and Caguas, 3 roads between Caguas and Ponce, and 4 roads between Ponce and Jurutungo. How many ways are there to travel between San Juan and Jurutungo?
4. In how many ways can a team of ten persons be divided into two teams of five.
5. In how many ways can 5 men and 5 women be seated in a row of 10 chairs if no two persons of the same sex are in adjacent chairs.
6. For each natural number  $k$  let  $\mathbb{N}_k$  denote the set  $\{n \in \mathbb{N} : 1 \leq n \leq k\}$ .
  - (a) Find the number of functions from  $\mathbb{N}_5$  to  $\mathbb{N}_{365}$ .
  - (b) Find the number of injective functions from  $\mathbb{N}_5$  to  $\mathbb{N}_{365}$ .
  - (c) Find the probability that in a set of 5 people there is a pair with the same birthday.
7. How many words of three letters can be formed from the alphabet  $\mathcal{A} = \{a, b, c, d, e, f\}$ ?
8. How many words of three letters containing exactly one  $e$  can be formed from the alphabet  $\mathcal{A} = \{a, b, c, d, e, f\}$ ?
9. A DNA chain is a word in the alphabet  $\{A, G, C, T\}$ . Find the number of words of length 14 with
  - (a) 4 A's and 10 G's.
  - (b) 4 A's and 5 G's, 2C's and 3T's
  - (c) C at one end and A or G at the other.
  - (d) C at one end and A or G at the other and with exactly 3 A's.?
  - (e) At least 12 consecutive A's.
10. A library contains 6 books in French, 5 in Spanish and 8 in English.
  - (a) How many ways can a subset of two books be chosen if they have different languages.
  - (b) How many ways can a subset of three books be chosen if exactly two are in French.