

MATH 3225 - Homework: Balls in Boxes

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1. How many n digit binary numbers are there containing exactly k zeros?
2. How many strictly increasing functions $f : \{1, 2, \dots, k\} \rightarrow \{1, 2, \dots, n\}$ are there?
3. In how many ways can n labelled balls be placed in 2 labelled boxes if the first box contains exactly k balls.
4. Find the number of integers between 0 and 999999 with digit sum equal 9.
5. Find the number of edge unlabelled graphs with vertex set $\{1, 2, \dots, n\}$ and m edges. (Loops and parallel edges are permitted).
6. If k different brands of beer are available in a store, how many ways can n beers be purchased?
7. In how many ways can k beers of the same brand be divided among n people.
8. In how many ways can k beers of the same brand be shared among n people if each person has at least one beer.
9. Find the number of solutions of the equation
$$x + y + z + w = 5$$
if $x, y, z, w \in \mathbb{N}$.
10. If f is an differentiable function of n variables and $k \in \mathbb{N}$, find the number of partial derivatives $D_1^{k_1} D_2^{k_2} \dots D_n^{k_n}$ of order k (i.e., such that $\sum_{i=1}^n k_i = k$.) You may assume that the order of differentiation is not important.
11. Let $n, m \in \mathbb{N}$. Find the number of paths of length $m + n$ on the lattice $\mathbb{Z} \times \mathbb{Z}$ which start at $(0, 0)$ and terminate at (n, m) .
12. Use the inclusion-exclusion principle to count the number of ways of distributing 7 distinguishable balls in 3 distinguishable boxes if each box contains at least 1 ball?

¹<http://pennance.us>