

Edge Labelled Graphs¹

1. An *e-labelled graph* G is a triple (V, E, ψ) where V is a nonempty set of *vertices*, E is a set of *edges* with $V \cap E = \emptyset$, and

$$\psi: E(G) \rightarrow \binom{V}{2} \cup V$$

is the *incidence function*.

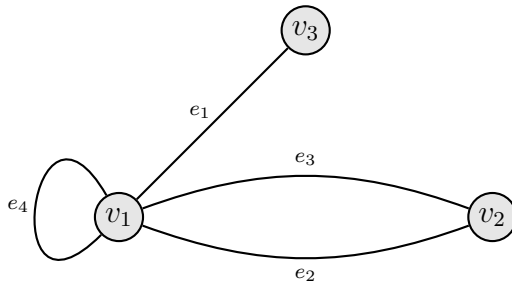
2. Let $G = (V, E, \psi)$ where

$$\begin{aligned} V(G) &= \{v_1, v_2, v_3\}, \\ E(G) &= \{e_1, e_2, e_3, e_4\}, \end{aligned}$$

and $\psi: E(G) \rightarrow \binom{V}{2} \cup V$ is given by

$$\begin{aligned} e_1 &\mapsto \{v_1, v_3\}, \\ e_2 &\mapsto \{v_1, v_2\}, \\ e_3 &\mapsto \{v_1, v_2\}, \\ e_4 &\mapsto v_1. \end{aligned}$$

Then G is an *e-labelled graph*. One drawing of G is:



3. Two e-labelled graphs G and H are *equal*, written $G = H$, if $V_G = V_H$, $E_G = E_H$ and $\psi_G = \psi_H$; otherwise $G \neq H$.

4. Let $G = (V, E, \psi)$ be an e-labelled graph.

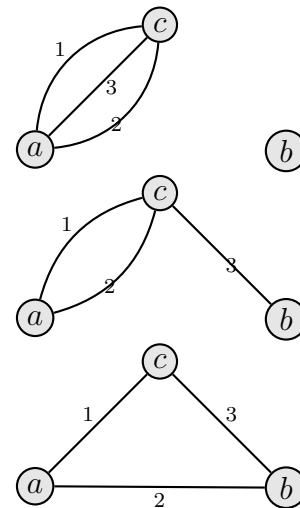
- (a) G has *no parallel edges* if ψ is injective.
- (b) G is *loopless* if $\text{Im } \psi \subseteq \binom{V}{2}$.
- (c) G is *simple* if it has neither loops nor parallel edges.
- (d) G is *complete* if $\text{Im } \psi = \binom{V}{2}$.

5. If $G = (V, E, \psi)$ is an *e-labelled simple graph*, then $|E| \leq \binom{|V|}{2}$.

6. Two e-labelled graphs G and H are *isomorphic* (written $G \approx H$) if there exist bijections $\alpha: V_G \rightarrow V_H$ and $\beta: E_G \rightarrow E_H$ such that

$$\psi_G(u) = \{x, y\} \iff \psi_H(\beta(u)) = \{\alpha(x), \alpha(y)\}.$$

7. Every loopless *e-labelled graph* with $V = \{a, b, c\}$ and $E = \{1, 2, 3\}$ is isomorphic to one of the three graphs below.



¹Partially based on a 1996 Lecture by A. Kelmans, Department of Mathematicse, University of Puerto Rico.