Geometry - Miscellaneous Problems

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1. Consider the $\triangle ABC$ in which

   $A = (1, 2, 3)$, $\quad B = (-1, -2, 2)$, and

   $C = (2, 3, -2)$.

   Find:

   (a) the cosine of the angle between the sides AB and AC

   (b) the length of the side AB.

   (c) the midpoint of side AB.

2. If a diagonal and an edge of a cube intersect. Find the angle between them.

3. Let a, b be non zero vectors in $\mathbb{R}^2$. If $\langle a, b\rangle = |a||b|$, show that $[a] = [b]$.

4. Let $a = (a_1, a_2) \in \mathbb{R}^2$. Define

   $$a^\perp = (-a_2, a_1)$$

   Verify that $\langle a, a^\perp\rangle = 0$.

5. Show that the sides of a parallelogram are all of equal length if and only if the diagonals are perpendicular.

6. Show that the sum of the squares of the sides of a parallelogram is equal to the sum of the squares of the diagonals.

7. Let a, b be non zero vectors in $\mathbb{R}^3$ and such that $\langle a, b\rangle = 0$. Let α, β and γ are real numbers such that $\alpha a + \beta b + \gamma a \times b = O$. Show $\alpha = \beta = \gamma = 0$.

8. Find the tangent line to the curve $\gamma(t) = (\cos 4t, \sin 4t, t)$ at the point $t = \pi/8$.

9. Let $a, b \in \mathbb{R}^2$. Show that a and b are linearly independent if and only if $a \wedge b \neq 0$.

10. Consider the affine space $(\mathbb{R}^2, \overrightarrow{\mathbb{R}^2}, +)$. Show that choice of origin $a \in \mathbb{R}^2$ sets up a one to one correspondence $f : \mathbb{R}^2 \to \overrightarrow{\mathbb{R}^2}$ between points and position vectors. Hint: Consider the functions $f : \mathbb{R}^2 \to \overrightarrow{\mathbb{R}^2}$ given by $f(x) = \overrightarrow{ax}$. Define $g : \overrightarrow{\mathbb{R}^2} \to \mathbb{R}^2$ by $g(\overrightarrow{u}) = a + \overrightarrow{u}$. Show that f is invertible with inverse g.

[1] http://pennance.us