Math 4009 - Homework: Variation of Parameters

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1. *a*) Find a basis for the solution space of the ordinary differential equation $x'' = 0$.

   b) Use the method of undetermined coefficients to find a general solution of the ordinary differential equation $x'' = t$.

   c) Use the method of variation of parameters to find a solution of ordinary differential equation $x'' = t$.

2. *a*) Find all solutions of:

   $$x'' - \frac{2}{t^2}x = 0; \quad 0 < t < \infty \tag{1}$$

   Hint: Try functions of the form $x = t^r$. How do you know you've found all the solutions?

   b) Use the method of variation of parameters to find all solutions of (**??**).

3. Find the general solution of:

   $$tx'' - (t+1)x' + x = t^2$$

   given that

   $$u_1(t) = e^t, \qquad u_2(t) = t + 1$$

   is a basis for the solution space of the corresponding homogeneous differential equation.

4. Show that the solution of the initial value problem:

   $$y'' + 2y' + 2y = f(t); \qquad y(0) = 0, \quad y'(0) = 0$$

   is

   $$y = \int_0^t e^{t-\tau} f(\tau) \sin(t-\tau) d\tau$$

5. Verify that $y_1(x) = e^{x^2+x}$ and $y_2(x) = e^{x^2-x}$ are linearly independent solutions of the homogeneous equation $y'' - 4xy' + (4x^2 - 3)y = 0$. Hence find the general solution of $y'' - 4xy' + (4x^2 - 3)y = e^{x^2}$

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