Math 4009 -Homework on Linear Second Order ODE's

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1. Find the general solution of each of the following differential equations.
   (a) $x'' = 0$
   (b) $x'' + x = 0$
   (c) $x'' - x = 0$
   (d) $x'' + 3x = 0$
   (e) $x'' - 2x' + x = 0$
   (f) $x'' + x' - 6x = 0$

2. Find the general solution of each of the following differential equations.
   (a) $x'' + x = e^t + t$
   (b) $2y'' + 12y' + 18y = e^{-3x}$
   (c) $2y'' + 3y' + y = 3\sin x$
   (d) $x'' + 4x = \cos 3t$
   (e) $x'' + x' + 4x = 2\sinh t$

3. Find the solution of each of the following initial value problems and sketch the graphs of the solution.
   (a) $\begin{cases} x'' + x' - 2x = 2t, \\ x(0) = 0, \quad x'(0) = 1 \end{cases}$
   (b) $\begin{cases} y'' + 4y = t^2 + 3e^x, \\ y(0) = 0, \quad y'(0) = 2 \end{cases}$
   (c) $\begin{cases} z'' + 8z' - 9z = 0 \\ z(0) = 1, \quad z'(0) = 0 \end{cases}$

4. Let k, m, γ be positive constants. State the general solution general solution of:
   (a) Free particle motion: $mx'' = 0$.
   (b) Spring equation: $mx'' + kx = 0$
   (c) Motion with retardation:
   $$mx'' + 2\gamma x' = 0$$

5. Consider the initial value problem
   $2y'' + 3y' - 2y = 0$
   $\begin{cases} y(0) = 0 \\ y'0) = -\beta \end{cases}$
   where $\beta > 0$.
   (a) Solve the initial value problem.
   (b) Plot the solution when $\beta = 1$. Find the coordinates (x_0, y_0) of the minimum point of the solution in this case.
   (c) Find the smallest value of β for which the solution has no minimum point.

[1] https://pennance.us