

Homogeneous Differential Equations

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1. Find implicit forms of the solutions for each of the following equations.

(a) $y' = \frac{1}{x} (y + \sqrt{x^2 + y^2})$.

(b) $y' = \frac{x^2 + xy + y^2}{x^2}$.

(c) $x' - \frac{x}{t} = \frac{1}{\ln(x^2) - 2 \ln t}$.

(d) $y' = \frac{2y - x + 5}{2x - y - 4}$.⁴

Hint: Substitute

$$y(\xi) := y(\xi + h) + k$$

and determine the constants h and k such that the transformed equation is homogeneous.

2. Find the general solution of the differ-

ential equation

$$y' = (x + y)/x.$$

Solution. $y = x(\ln |x| + C)$.

3. Show that

$$y' - \frac{y}{x} = \frac{1}{\ln(y^2) - 2 \ln x}$$

is a homogeneous equation and find its general solution in implicit form.

4. Let $X' = F(t, x)$ be homogeneous. Show that if $\mu(t)$ is a solution of the (separable) differential equation:

$$M' = \frac{F(1, M) - M}{t},$$

then $\phi(t) = t\mu(t)$ is a solution of the homogeneous equation determined by F .

¹<https://pennance.us>