

Relations in Graphic Matroids¹

1. Definition.

Let E be a set and let $\mathcal{A} \subseteq 2^E$. The following notation will be used for certain subsets of 2^E

- (a) $\neg \mathcal{A} = \{X \in 2^E : X \notin \mathcal{A}\}$.
- (b) $\perp \mathcal{A} = \{X \in 2^E : |X \cap A| \neq 1, \forall A \in \mathcal{A}\}$.
- (c) $\mathbf{cs} \mathcal{A} = \{X \in 2^E : X \text{ contains some } A \in \mathcal{A}\}$.
- (d) $\mathbf{cn} \mathcal{A} = \{X \in 2^E : X \text{ contains no } A \in \mathcal{A}\}$.
- (e) $\mathbf{cis} \mathcal{A} = \{X \subseteq E : X \text{ is contained in some } A \in \mathcal{A}\}$.
- (f) $\mathbf{cin} \mathcal{A} = \{X \subseteq E : X \text{ is contained in no } A \in \mathcal{A}\}$.
- (g) $\mathbf{Max} \mathcal{A} = \{A \in \mathcal{A} : A \text{ is maximal in } \mathcal{A}\}$.
- (h) $\mathbf{Min} \mathcal{A} = \{A \in \mathcal{A} : A \text{ is minimal in } \mathcal{A}\}$.
- (i) $\mathbf{Comp} \mathcal{A} = \{X \subseteq E : E \setminus X \in \mathcal{A}\}$.

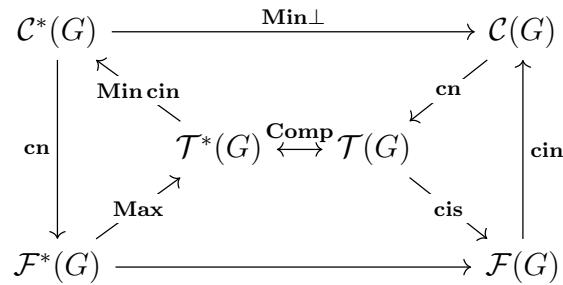
2. Claim.

Let G be an e-labelled graph with edge set $E = E(G)$. Any of the following lists describes the matroid $M(G)$ of G .

- (a) $\mathcal{C}(G)$ – cycles of G .
- (b) $\mathcal{C}^*(G)$ – cocycles (minimal cuts).
- (c) $\mathcal{T}(G)$ – frames (maximal forests).
- (d) $\mathcal{T}^*(G)$ – cotrees of G .
- (e) $\mathcal{F}(G)$ – forests (acyclic).
- (f) $\mathcal{F}^*(G)$ – co-forests (a-cocyclic).

3. Some Relations.

- (a) $\mathcal{F}(G) = \mathbf{cn} \mathcal{C}(G)$.
- (b) $\mathcal{T}(G) = \mathbf{Max}(\mathcal{F})$.
- (c) $\mathcal{F}(G) = \mathbf{cis} \mathcal{T}(G)$.
- (d) $\mathcal{C}(G) = \mathbf{Min} \mathbf{cin} \mathcal{T}(G)$.
- (e) $\mathcal{T}(G) = \mathbf{Comp} \mathcal{T}^*(G)$.
- (f) $\mathcal{F}^*(G) = \mathbf{cn} \mathcal{C}^*(G)$.
- (g) $\mathcal{T}^*(G) = \mathbf{Max} \mathcal{F}^*(G)$.
- (h) $\mathcal{F}^*(G) = \mathbf{cis} \mathcal{T}^*(G)$.
- (i) $\mathcal{C}^*(G) = \mathbf{Min} \mathbf{cin} \mathcal{T}^*(G)$.
- (j) $\mathcal{T}^*(G) = \mathbf{Comp} \mathcal{T}(G)$.



¹Notes by Philip Pennance – based on Lectures by Alexander Kelmans at the University of Puerto Rico, circa 1996.