Axioms for Matroid Theory[1]

Axioms AI

AI1 $\mathcal{I} \neq \emptyset$.

AI2 Hereditary Property

If $X \in \mathcal{I}$ and $Y \subseteq X$ then $Y \in \mathcal{I}$.

AI3 Augmenting Property

If $X,\ Y \in \mathcal{I}$ and $|X| < |Y|$ then
$\exists y \in Y \setminus X$ such that $X + y \in \mathcal{I}$.

Axioms A′I

A′I1 $\mathcal{I} \neq \emptyset$.

A′I2 Hereditary Property

If $X \in \mathcal{I}$ and $Y \subseteq X$ then $Y \in \mathcal{I}$.

A′I3 Basis equicardinality.

If $B, B' \in \mathcal{B}$ then $|B| = |B'|$.

Axioms AB

AB1 $\mathcal{B} \neq \emptyset$.

AB2 Clutter

For all $B, B' \in \mathcal{B}$,
if $B \neq B'$ then $B \not\subset B'$.

AB3 Exchange

If $X, Y \in \mathcal{B}$, and $x \in X \setminus Y$ there exists $y \in Y \setminus X$ such that $X + y - x \in \mathcal{B}$.

Axioms A′B

A′B1 $\mathcal{B} \neq \emptyset$.

A′B2 Basis equicardinality.

If $B, B' \in \mathcal{B}$ then $|B| = |B'|$.

AB3 Exchange

If $X, Y \in \mathcal{B}$, and $x \in X \setminus Y$ there exists $y \in Y \setminus X$ such that $X + y - x \in \mathcal{B}$.

Axioms AC

AC1 $\emptyset \notin \mathcal{C}$.

AC2 Clutter

For all $C, C' \in \mathcal{C}$,
if $C \neq C'$ then $C \not\subset C'$.

AC3 Weak Circuit Elimination

For all $X, Y \in \mathcal{C}$, and $a \in X \cap Y$
there exists $Z \in \mathcal{C}$ such that
$Z \subseteq X \cup Y - a$.

Axioms A′C

A′C1 $\emptyset \notin \mathcal{C}$.

A′C2 Clutter

For all $C, C' \in \mathcal{C}$,
if $C \neq C'$ then $C \not\subset C'$.

A′C3 Strong Circuit Elimination

For all $X, Y \in \mathcal{C}$, and $a \in X \cap Y$ and $x \in X$
there exists $Z \in \mathcal{C}$ containing x such that
$Z \subseteq X \cap Y - a$

Axioms AIC

AIC1 $\mathcal{I} \neq \emptyset$

AIC2 Hereditary Property

If $X \in \mathcal{I}$ and $Y \subseteq X$ then $Y \in \mathcal{I}$.

AIC3 Fundamental Circuit Axiom

For all $X \in \mathcal{I}$, if $a \in E \setminus X$ there exists at most one circuit $C \subseteq X + a$ containing a.

Axioms A⋆B

A⋆B1 $\mathcal{B} \neq \emptyset$

A⋆B2 Clutter
For all $B, B' \in \mathcal{B}$,
if $B \neq B'$ then $B \not\subset B'$.

A⋆B3 Exchange⋆

If $X, Y \in \mathcal{B}$, and $y \in Y \setminus X$ there exists $x \in X \setminus Y$ such that $X + y - x \in \mathcal{B}$.

[1]Notes by Philip Pennance – based on Lectures by Alexander Kelmans at the University of Puerto Rico, cerca 1996.