

MATH 3225 - Homework on Functions

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1. Let

$$f(x) = (x - 3)^2, \quad x \in \mathbb{R}$$

and

$$h(x) = x^2, \quad x \in \mathbb{R}$$

Show that the graph of h is a translation of the graph of f .

2. Let $f : [0, 4] \rightarrow \mathbb{R}$ such that $x \mapsto \sqrt{x}$.

(a) Show that f is injective.

(b) Find two left inverses of f .

(c) Let $h(x) = x^2$, $x \in \mathbb{R}$. Show that h is not a left inverse of f .

(d) Show that f has no inverse function.

3. (a) Show that the composition of two injections is an injection.

(b) Show that the composition of two surjections is a surjection.

4. Let $f : A \rightarrow B$ be a bijection. Let g_L be any left inverse of f , g_R any right inverse of f and g any inverse of f . Show that $g_L = g_R = g$.

5. Find a set B such that the triple $F = (\mathbb{R}, B, \{(x, x^2 - 2x) : x \in \mathbb{R}\})$ is a surjective function. Find a right inverse of F .

6. Find a set $A \subseteq \mathbb{R}$ such that the triple $(A, \mathbb{R}, \{(x, x^2 - x - 6) : x \in A\})$ is an injective function. Find a left inverse of F .

7. Find sets $A \subseteq \mathbb{R}$ and $B \subseteq \mathbb{R}$ such that the triple $F = (A, B, \{(x, 4 \sin 2x) : x \in A\})$ is a bijective function. Find the inverse function of F and draw its graph.

8. Find a set $A \subseteq \mathbb{R}$ such that the triple $(A, \mathbb{R}, \{(x, 5) : x \in A\})$ is an injective function. Find a left inverse of F .

9. Let S be the set of circles in the plane and let $f : S \rightarrow \mathbb{R}$ be defined by $f(S) =$ the area of S . Is f injective?, surjective?, bijective?.

Now let T be the set of circles in the plane whose center is the origin and define $g : T \rightarrow \mathbb{R}^+$ by $g(T) =$ the length of the circumference of T . Is g injective? surjective? bijective?. Justify your answers.

10. Let $\mathbb{Z}$ be the set of integers and $\mathbb{Q}$ the set of rational numbers. Find all triples a, b, c of rational numbers such that $(\mathbb{Z}, \mathbb{Z}, \{(x, ax^2 + bx + c) : x \in \mathbb{Z}\})$ is a function.

11. Let $\mathbb{Z}$ be the set of integers and $\mathbb{Q}$ the set of rational numbers. Let $G = \{(m/n, m) : m, n \in \mathbb{Z}, n \neq 0\}$. Is the triple $(\mathbb{Q}, \mathbb{Z}, G)$ a function? (Justify your answer)

12. Let $G = \{(\sqrt{x}, \sin x) : x \in [0, 4]\}$. Is the triple $([0, 2], \mathbb{R}, G)$ a function? Prove your answer.

13. Let $G = \{(\sin x, x^2) : x \in [0, \pi]\}$. Show that the triple $([0, 1], \mathbb{R}, G)$ is not a function.

14. Let

$$G = \{(\sin x, (x - \pi/2)^2) : x \in [0, \pi]\}.$$

Show that the triple $([0, 1], \mathbb{R}, G)$ is a function.

¹<https://pennance.us>

15. Let $f : A \rightarrow B$ and $g : A \rightarrow C$ and $G = \{(f(t), g(t)) : t \in A\}$. Find sufficient conditions for the triple (B, C, G) to be a function.
16. Let $f : A \rightarrow B$ and $g : B \rightarrow C$. Show that $g \circ f$ surjective implies that g surjective.
17. If $f : A \rightarrow B$ and $g : B \rightarrow A$ satisfy $f \circ g = id_B$ show that f is surjective and g is injective.
18. Let $f : A \rightarrow B$ and $g : B \rightarrow C$. Show that $g \circ f$ surjective implies that g surjective.
19. Let $f : A \rightarrow B$ and $g : B \rightarrow C$. Show that $g \circ f$ surjective and g injective imply that f is surjective.
20. Let f be a function with finite domain and codomain. If the cardinality of the domain equals the cardinality of the codomain, is it true that if f is not injective, then f must be non-surjective as well? Prove or disprove.
21. Let $f : \mathbb{Z} \times (\mathbb{Z} \setminus 0) \rightarrow \mathbb{Q}$ be the function with values given by $f(m, n) = m/n$. Let $g : \mathbb{Z} \times (\mathbb{Z} \setminus 0) \rightarrow \mathbb{Z}$ be the function with values given by $g(m, n) = m$. Let $G = \{(f(m, n), g(m, n)) : (m, n) \in \mathbb{Z} \times (\mathbb{Z} \setminus 0)\}$. Show that the triple $(\mathbb{Q}, \mathbb{Z}, G)$ is not a function.
22. Let $f = (A, B, G_f)$ and $g = (B, C, G_g)$ are functions. Let $g \circ f = (A, C, G)$ where $G = \{(a, c) \in A \times C : (a, b) \in G_f, \text{ and } (b, c) \in G_g \text{ for some } b \in B\}$.
 - (a) Show that $g \circ f$ is a function.
 - (b) Show that $g \circ f(a) = g(f(a))$ for all $a \in A$.
 - (c) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ and $h : C \rightarrow D$ be functions. Show that $(h \circ g) \circ f = h \circ (g \circ f)$.