Math 3153 - Optimization: Functions of Several Variables

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1. Let $g(x,y) = \frac{1}{2}(ax^2 + 2bxy + cy^2)$. Show, by completing the square, that $g \geq 0$ if and only if $ac - b^2 > 0$ and $a > 0$.

2. Let $f(x,y) = 2x^2 + y^2 - xy - 7y$.

   (a) Show that $(1,4)$ is the only critical point of f.

   (b) Test the critical point by means of the Hessian.

   (c) Verify your calculation in (b) by examining $f(1+h, 4+k) - f(1,4)$ for h, k small.

3. Find the absolute maxima and minima of the function $f(x,y) = x^2 - xy + y^2$ in the rectangle $S = [-1,1] \times [-1,1]$.

4. Compute the second degree Taylor approximation

$$f(p+h) = f(p) + \langle h, \nabla \rangle f(p) + \frac{1}{2!} \langle h, \nabla \rangle^2 f(p)$$

   at $p = (2,1)$ for the function $f(x,y) = 1 + x + 2y^2 + x^3$.

5. Compute the second degree Taylor expansion at $p = (0,0)$ for the function $f(x,y) = e^x \cos y$.

6. Let $f(x,y) = (x-1)^2 + y^2$.

   (a) Find the critical points of f and test them by means of the Hessian.

   (b) Find the Taylor expansion of f of order 2 about the origen.

[1] https://pennance.us