

UNIVERSITY OF PUERTO RICO – RÍO PIEDRAS
DEPARTMENT OF MATHEMATICS

PRACTICE QUESTIONS FOR EXAM II
Prof. Philip Pennance

1. Sketch the levels curves of height c

$$f(x, y) = \ln(x) - \ln(y)$$

for all

$$c \in \{-\ln(3), -\ln(2), 0, \ln(2), \ln(3)\}$$

2. Let

$$u(x, y) = e^x \cos y$$

$$v(x, y) = e^x \sin y.$$

Show that

$$D_1 u(x, y) = D_2 v(x, y)$$

$$D_2 u(x, y) = -D_1 v(x, y)$$

for all $(x, y) \in \mathbb{R}$

3. Let $f(x, t) : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $f(x, t) = \sin(x - ct)$. Show that f satisfies the wave equation:

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}.$$

4. Let $n \in \mathbb{R}^3$ a unit vector. For each $t \in \mathbb{R}$ define a function $f_t : \mathbb{R}^3 \rightarrow \mathbb{R}$ by $f_t(x) = \sin(n \cdot x - 3t)$. Show that f_t is constant on any plane of the form $\vec{n} \cdot \vec{x} = d$. Give a physical interpretation.

5. Let

$$f(x, y, z) = \ln zx + \cos xy + e^{yz}.$$

Find the directional derivative $D_u f(P)$ at the point $P = (\frac{1}{3}, 0, 3)$ in the direction $\vec{u} = (1, 2, 3)$.

6. Let $f(x, y) = x^2 + y$. Find an equation of the tangent plane to the graph of f at the point $(1, 2, 3)$.

7. Find $\frac{du}{dt}$ if

$$u(x, y) = x^2 - y^2$$

$$x(t) = \cos t$$

$$y(t) = \sin t.$$

8. Find the minimum and maximum values of

$$f(x, y) = 3x + 4y$$

subject to the constraint $x^2 + y^2 = 1$.

9. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be given by

$$f(x, y) = (2xy, x^2 + 3y^2).$$

Find a function ϕ such that $f = \nabla \phi$.

10. Let $f(x, y) = 3x + 2y$. Find

(a) The coefficients a and b such that $Df(x, y)(h, k) = ah + bk$

(b) The directional derivative of f at $(1, 1)$ in the direction $(3, 4)$.

(c) A normal vector to the graph of f at the point $(1, 1, 5)$.

11. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be given by $f(x, y) = (2xy, x^2 + 3y^2)$. Find a function ϕ such that $f = \nabla \phi$.

12. Find and test, by means of the Hessian, the critical point(s) of

$$f(x, y) = 2x^2 + y^2 - xy - 7y.$$

13. Compute the second degree Taylor expansion at $p = (0, 0)$ for the function

$$f(x, y) = e^x \cos y.$$

14. Find the maximum of the entropy function

$$H(p_1, p_2, p_3, p_4) = - \sum_{i=1}^4 p_i \ln p_i$$

subject to the constraint $p_1 + p_2 + p_3 + p_4 = 1$.

15. Let $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be given by $L(x, y) = (L_1(x, y), L_2(x, y))$. If L is linear, show that the component functions L_1, L_2 are linear.

16. Show that

$$\nabla \frac{1}{\|X\|} = - \frac{X}{\|X\|^3}.$$

17. (a) Let $f(x) = 2x$. Find $Df(3)(2)$
 (b) Let $f(x, y) = (2x+3y, 4x+5y)$ and $\alpha, \beta \in \mathbb{R}$. Find $Df(1, 1)(\alpha, \beta)$.

18. Find a degree 2 polynomial approximation of the function

$$f(x, y) = \log(1 + x + 2y)$$

around $P = (2, 1)$.

19. Example.

Find the extreme points of

$$f(x, y, z) = xyz$$

in the first octant subject to the constraint

$$xy + yz + zx = 2.$$

20. Let $f(x, y) = \frac{y \ln(x + \sqrt{x^2 - 1})}{x^2 + y^2}$.

Evaluate $\frac{\partial f}{\partial y}(1, 4)$.

21. (a) Let U be an open subset of $\mathbb{R}^2$ and $f : U \rightarrow \mathbb{R}$. Let (a, b) be a point in U . Define the partial derivative $\frac{\partial f}{\partial x}(a, b)$.

- (b) Evaluate $\frac{\partial f}{\partial y}(2, 3)$ if

$$f(x, y) = \frac{\sin(x - 2)}{1 + \sin(y^2 + \tan xy)}$$

- (c) Evaluate $\frac{\partial f}{\partial x}(0, 0)$ if

$$f(x, y) = \sqrt{xy}.$$

22. Let

$$f(x, y) = \frac{x + y}{1 - xy}.$$

Evaluate $D_2 f(2, 3)$.