Department of Mathematics

Math 3153-Calculus III - Presential
Semester II, 2022-23

Time:
L W 2:30 pm - 3:20 pm Location: CNL-A-207
M J 2:30 pm - 3:50 pm Location: CNL-A-207

Instructor: Philip Pennance
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Office hours: M,T,W,Th 10AM-12AM or by appointment.

Course page:

Catalog Description

MATH 3153. Calculus III. *Four credits. Four hours of lecture and one hour of discussion per week. Prerequisite: MATH 3152.*

Vectors. Vector calculus. Functions of several variables. Gradients. Extreme values. Differentials. Double and triple integrals. Line and surface integrals.

Course Objectives

After this course the students are expected to:

1. be competent in the methods of mathematical reasoning and proof relating to each part of the syllabus.
2. have developed their skills in thinking logically, formulating precise mathematical arguments, solving problems and presenting solutions in a good mathematical style.
3. have become familiar with the definitions, concepts, results and methods of proof relating to each part of the syllabus.
4. be able to quote the definitions and results, and to reproduce the proofs of some key results.
5. be able to solve problems relating to the material covered. These might be straightforward applications of the definitions and results but might also be problems of a more testing nature.

Syllabus and Approximate Time Distribution[1]

Vectors (2 weeks)

(Lang Chapter I)

1. Vector algebra in 2 and 3 dimensions.
2. $\mathbb{R}^n$ as an Euclidean Space
   (a) The scalar (inner, dot) product.
   (b) Orthogonality and projection.
   (c) Cauchy-Schwartz and triangle inequalities.
   (d) The cross product.
3. Lines in $\mathbb{R}^3$ Sectioning of a line segment, the distance between two skew lines.
4. Planes in $\mathbb{R}^n$, the distance from a point to a plane.
5. Characterization of linear maps in Euclidean-Space.
6. Determinants and orientation in $\mathbb{R}^2$.
7. The *signed area* of a parallelogram:

$$A^s = \begin{cases} A(P(u,v)) & \text{if } D(u,v) \geq 0, \\ -A(P(u,v)) & \text{if } D(u,v) < 0 \end{cases}$$
$$= D(u,v).$$

Curves (2 weeks)

(Lang Chapter II)

1. Limits, continuity and differentiability of curves.
2. Speed and acceleration.
3. Tangent and normal lines to a differentiable curve.
4. The arclength of a C^1 path $\gamma : [a,b] \to R^n$:

$$l(\gamma) = \int_a^b \|\gamma'(t)\|\, dt.$$

5. Unit speed reparametrization.
   (a) Curvature of a path γ parametrized by arclength:
   $$k(s) = \|\gamma''(s)\|.$$
   (b) Curvature of a path γ with parameter t.
   $$k(t) = \frac{\|\gamma'(t) \times \gamma''(t)\|}{\|\gamma'(t)\|^3}\ .$$

Functions of Several Variables (1 week)

(Lang Chapter III)

1. Scalar and vector valued functions.
2. Graphs and level curves.
3. Limits, continuity.

Differentiability (2 weeks)

(Lang, Chapters IV and XI.1)

1. Reformulation of the derivative in one dimension.
2. Matrices.
3. The derivative as a linear map.
4. The derivative of a scalar valued function. Directional and partial derivatives.
5. The derivatives of linear and bilinear maps.
6. The chain rule (special case).
   Let U be an open subset of $\mathbb{R}^2$ and
   $$f : U \subseteq \mathbb{R}^2 \to \mathbb{R}.$$
   differentiable. If γ a differentiable curve in U, then $f \circ \gamma$ is differentiable and
   $$\frac{d}{dt}(f \circ \gamma)(t) = \text{grad} f(\gamma(t)) \cdot \gamma'(t).$$
7. The derivative of a vector valued function. Jacobians.

[1] All right is reserved to adjust both time and content.

Curve Integrals (1 week)

(Lang Chapter V)

1. The line integral of a scalar function.
2. The line integral of a continuous vector field $\mathbf{F}$ along a C^1 path $\gamma : [a,b] \to \mathbb{R}^3$
$$\int_\gamma \mathbf{F} \cdot \mathbf{dr} = \int_a^b \mathbf{F}(\gamma(t)) \cdot \gamma'(t) dt.$$
3. Differentiation under the integral sign.

   Let $f : [a,b] \times [c,d] \to \mathbb{R}$. If f is continuous and $D_2 f$ exists and is continuous on $[a,b] \times [c,d]$, then
$$\frac{d}{dy} \int_a^b f(x,y)dx = \int_a^b D_2 f(x,y)dx.$$
4. Conservative fields.
5. Let F be a continuous vector field defined on an open path-connected set U. The following are equivalent:
   (a) $\mathbf{F}$ has a potential function defined on all of U.
   (b)
$$\int_\gamma \mathbf{F} \cdot \mathbf{dr} = 0.$$
   for all closed, piecewise C^1 paths γ.
   (c)
$$\int_{[P,Q],\gamma} \mathbf{F} \cdot \mathbf{dr}$$
   is independent of path.
6. Vector Fields and Differential Equations.

Higher Derivatives (1 week)

(Lang Chapter VI)

1. Claim (Equality of the partial derivatives).
   Let $F : U \subseteq \mathbb{R}^2 \to \mathbb{R}^2$ be a conservative vector field with components f, g. If $D_1 f, D_2 f, D_1 g, D_2 g$ are continuous and F is conservative then
$$D_2 f = D_1 g.$$
2. Taylor's theorem in one and higher dimensions.

Optimization (1 week)

(Lang Chapter VI)

1. Extrema of real valued functions.
2. Quadratic forms in 2 variables.
3. Let
$$g(h) = \frac{1}{2}(h_1, h_2) \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix},$$
   then
   (a) g is positive definite if and only if $ac - b^2 > 0$ and $a > 0$.
   (b) g is negative definite if and only if $ac - b^2 > 0$ and $a < 0$.
4. The Hessian:
$$\mathcal{H}_P(H) = \frac{(H \cdot \nabla)^2}{2!} f(P).$$
5. Sufficient conditions for local maxima and minima of a class C^3 function $f : U \subseteq \mathbb{R}^2 \to \mathbb{R}$ at a critical point.
6. Lagrange multipliers.

Multiple Integrals (1 week)

(Lang Chapter XII)

1. Double integrals.
2. Iterated integrals.
3. Polar coordinates
$$X(r,\theta) = (r\cos\theta, r\sin\theta).$$
4. Triple integrals.

Change of Variables Formula (1 week)

(Lang Chapter XIII)

1. Signed volume and determinants.
2. The Jacobian matrix.
3. The change of variables formula.

$$\iint_{X(R)} f = \iint_R f\circ X(u,v)|\det DX(u,v)|\,du\,dv.$$

4. Cylindrical coordinates:

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \end{aligned}$$

5. Spherical coordinates.

$$\begin{aligned} x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\phi\sin\theta \\ z &= \rho\cos\phi \end{aligned}$$

Green's Theorem (1 week)

(Lang Chapter XIV)

1. Green's theorem.

$$\iint_\Omega \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \int_{\partial\Omega} (P\,dx + Q\,dy).$$

2. Application to the Change of Variable Formula.

Surface Integrals (2 week)

(Lang Chapter XV)

1. The *area* of smooth parametrized surface S determined by a parametrization

$$X : U \subseteq \mathbb{R}^2 \to \mathbb{R}^3$$

$$\iint_S 1 d\sigma = \iint_U \|X_u \times X_v\|\,du\,dv.$$

2. Integrals of a scalar function over a smooth parametrized surface S :

$$\iint_S f d\sigma = \iint_U f(X(u,v))\|X_u \times X_v\|\,du\,dv.$$

3. Integral of a vector field $\mathbf{F}$ over a smooth parametrized surface S :

$$\iint_S (\mathbf{F}\cdot\mathbf{n})\,d\sigma = \iint_U \mathbf{F}(X(u,v))\cdot(X_u \times X_v)\,du\,dv.$$

4. Gauss' Divergence Theorem.

$$\iiint_V (\nabla\cdot\mathbf{F})\,dV = \iint_S (\mathbf{F}\cdot\mathbf{n})\,d\sigma.$$

5. Classic Stokes' theorem.

$$\iint_S \nabla\times\mathbf{F}\cdot\mathbf{n}\,d\sigma = \oint_{\partial S} \mathbf{F}\cdot d\mathbf{r}.$$

6. Applications of Stokes' Theorem.

Texts

1. Serge Lang, Calculus of Several Variables, Addison Wesley, 1968. (The course will follow the 1968 edition quite closely)
2. David A. Santos, *Multivariable and Vector Calculus*, Open Math Text, 2015. - (legally free download)

Bibliography

1. Saturnino L. Salas and Einar Hille, Calculus: One and Several Variables.
2. Jerrold Marsden and Anthony Tromba, Vector Calculus.

3. Serge Lang, A Second Course in Calculus.
4. Michael Spivak, Calculus.
5. Michael Spivak, Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus. (Very good book, though too advanced for this course).
6. Harold M. Edwards, Advanced Calculus: A Differential Forms Approach.
7. David A. Santos, *Pre- y Cálculo Criollos*, Open Math Text, 2008.
8. Ian Stewart and David Tall, *Foundations of Mathematics*, Oxford University Press, 1977.

Instructional Strategies and Policies

Instructional strategies: lectures, readings, exercises, exams and online videos.

Contingency Plan in Case of Emergency

In the event of a closure for any reason, the course will be continued via Google Meet.

Grading Scale

Letter grade (A, B, C, D or F)

Evaluation

Your grade will be based upon three partial exams (X_1, X_2, X_3), where $0 \leq X_i \leq 100$ and one comprehensive final exam F where $0 \leq X_F \leq 100$. The final average $\bar{X}$ will be calculated by eliminating the lowest grade, and giving double weight to the final. I.e.,

$$\bar{X} = \frac{(X_1 + X_2 + X_3 + 2 * X_F) - \min(X_1, X_2, X_3, X_F)}{4}.$$

The grading scale will be no worse than the following: A: $90 - 100$; B: $80 - 89$; C: $65 - 79$; D: 50-64; F: < 50. Cooperation with entities, human or electronic, if detected on exams and quizes, will result in an automatic F for the course. Calculators and electronic aparati may not be used. Use of calculators, computers, ipads, or any electronic devices in exams and quizzes is strictly forbidden. In accord with UPR regulations, persistent lateness or unexcused absence from class may result in a lowered or failing grade and loss of financial support.

You should attempt as many of the assigned exercises as needed to understand each topic. Quizes and exams will be heavily based on the assigned exercises so it is important to attempt as many as possible.

According to UPR regulations, the following items must be included in All syllabi.

Academic Honesty

All homework should be done independently –collaboration is not permitted. Cheating and other anti-intellectual behavior may result in an F. Please make sure you read, understand and abide by the Academic Integrity Code of the University of Puerto Rico.

Students with Disabilities

If you have a disability for which you may be requesting an accommodation, please contact both your instructor and the office of Vocational Rehabilitation as early as possible in the term. Vocational Rehabilitation will verify your disability and determine reasonable accommodations for this course.

Regulations on discrimination (Certification 39, 2018-2019)

"The University of Puerto Rico prohibits discrimination based on sex, sexual orientation, or gender identity in any of its forms, including that of sexual harassment. According to the Institutional Policy Against Sexual Harassment at the University of Puerto Rico, Certification Num. 130, 2014-2015 from the Board of Governors, any student subjected to acts constituting sexual harassment, must come to the Office of the Student Ombudsperson, the Office of the Dean of Students, and/or the Coordinator of the Office of Compliance with Title IX for an orientation and/or a formal complaint".