Some Different Ways of Defining Functions

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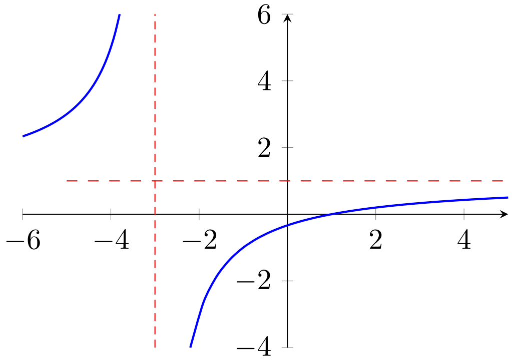
1. Finite function $f = (D_f, C_f, G_f)$ with domain, codomain and graph specified by extension:

$$D_f = \{1, 2, 3\}$$
$$C_f = \{1, 4, 9, 12\}$$
$$G_f = \{(1, 1), (2, 4), (3, 9)\}$$

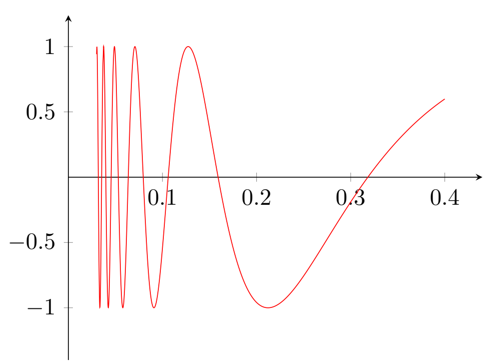
2. Function with graph determined by an algebraic expression:

$$f(x) = \frac{x-1}{x+3} \qquad (x \neq 3)$$



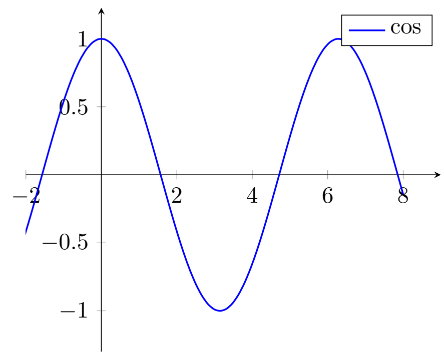
3. Function determined by cases:

$$f(x) = \begin{cases} \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$$



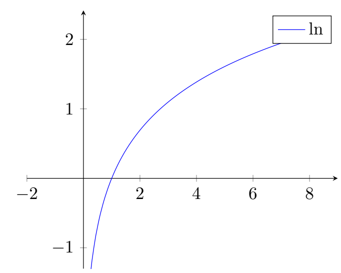
4. Function determined by an initial value problem:

$$f'' + f = 0$$
$$f(0) = 1$$
$$f'(0) = 0$$



5. Function determined by the fundamental theorem of calculus:

$$\ln(x) = \int_1^x \frac{1}{t} dt$$



6. Function determined implicitly:

$$f(x) + \cos^2[f(x)] = x$$

7. Function determined by recursion:

$$f(n) = 2f(n-1)$$
$$f(0) = 1$$

8. Function determined by a series:

$$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

9. Function determined by its inverse.

$$\cosh = A_h^{-1}$$

where

$$A_h(\xi) = \xi\sqrt{\xi^2 - 1} - 2\int_1^{\xi} \sqrt{x^2 - 1}dx$$

[1] https://pennance.us