Reflection and Refraction

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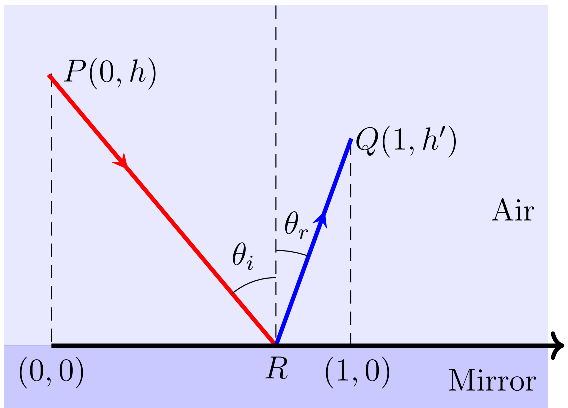
1. Law of Reflection.

   When light is reflected off a plane surface, the angle of incidence θ_i is equal to the angle of reflection θ_r.

   Proof.



   By Fermat's Principle, light follows the path of least time. Hence we seek R such that

$$L = d(P,R) + d(R,Q)$$

   is a minimum. Choose coordinates as shown and let $R = (x,0)$. We seek x which minimizes

$$L(x) = \sqrt{x^2 + h^2} + \sqrt{(x-1)^2 + h'^2}$$

   Differentiating yields

$$L'(x) = \frac{x}{\sqrt{x^2 + h^2}} + \frac{x-1}{\sqrt{(x-1)^2 + h'^2}}$$

   Thus x is a critical point if and only if:

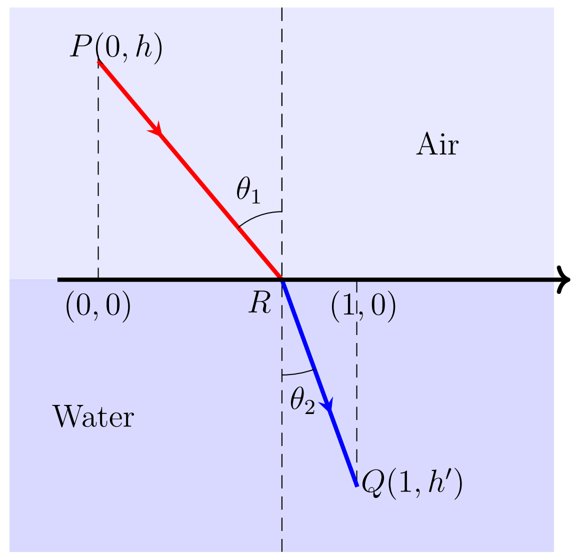
$$\begin{aligned} L'(x) = 0 &\Leftrightarrow \frac{x}{\sqrt{x^2+h^2}} = \frac{1-x}{\sqrt{(1-x)^2 + h'^2}} \\ &\Leftrightarrow \sin\theta_i = \sin\theta_r \\ &\Leftrightarrow \theta_i = \theta_r \end{aligned}$$

2. Snell's Law of Refraction.

   When light is refracted at the interface between the air and water, the angles of incidence and refraction (see diagram) satisfy

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

   where v_1 and v_2, respectively, are the velocity of light in air and water.



   Proof. Exercise.

3. Remark. The paths of ants are known to follow Snell's law at the interface between two types of terrain.

4. Exercise. Suppose that at time $t = 0$ a life guard is located at point $(0,0)$ in the (x,y)-plane. A swimmer is drowning at point (a,b) in the first quadrant of the same coordinate system. If speed of the life guard is

$$v(x) = \begin{cases} v_1 & \text{if } 0 \le x \le c, \\ v_2 & \text{if } c < x \le a. \end{cases}$$

   How can the lifeguard rescue the swimmer in minimum time? Justify your answer.

[1] http://pennance.us