

Math 3151 -Implicit Derivatives and Rate of Change Problems

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1. A street light is 10 ft tall. A 5 ft tall man is walking away from the pole at a rate of 2 ft s^{-1} .

(a) Find the speed of the shadow of the mans head when he is 20 ft from the pole?

(b) At what rate is at which the shadow of the mans head is moving away from the man he is 20 ft from the pole?

2. Let $x(t)$ be the position of a point in rectilinear motion at time t . Use the formula $|x| = \sqrt{x^2}$ and the chain rule to find the derivative $\frac{d}{dt}|x(t)|'$ for a sinusoidal motion $x(t) = \sin t$.

3. The coordinate of a point undergoing rectilinear motion is given by

$$x(t) = -t^2 + 4t + 5.$$

Find:

(a) The time τ at which the instantaneous velocity is zero.

(b) The set

$$\{t \in \mathbb{R} : x(t) \text{ is speeding up}\}.$$

(c) The maximum of the set

$$\{x(t) : t \in \mathbb{R}\}.$$

(d) The time t_0 guaranteed by the mean value theorem at which the instantaneous velocity $v(t_0)$ is equal to the average speed over the interval $[2, 10]$.

4. Find the equation of the tangent and normal line to the curve

$$x^2 - 4y^2 = 9$$

at the point $(5, 2)$.

5. Find the equation of the tangent and normal lines to the curve $\sqrt{x+y} + x + y = 12$ at the point $(5, 4)$.

6. A point is moving on the circle

$$(x - 1)^2 + y^2 = 25.$$

At the point $(5, -3)$ the x -coordinate is increasing at the rate of 5 cm s^{-1} Find the rate of change of the y -coordinate at the same moment.

7. Consider the curve $x = \cos y$.

(a) Find the slope of the curve at the point $(\frac{1}{2}, \frac{7\pi}{3})$.

(b) Write an equation for the normal line at the point $(\frac{1}{2}, \frac{7\pi}{3})$.

8. Show that the slope $m(x, y)$ of the curve $x = \sin y$ at the point (x, y) is given by

$$m(x, y) = \sec y.$$

9. Suppose that $y = f(x)$. If

$$2xy + \pi \sin y = 2\pi.$$

Find:

(a) An expression for $y' = \frac{dy}{dx}$ in terms of x, y .

(b) An expression for $y'' = \frac{d^2y}{dx^2}$ in terms of x, y .

(c) An equation for the tangent line to the graph of f at the point $(1, \frac{\pi}{2})$.

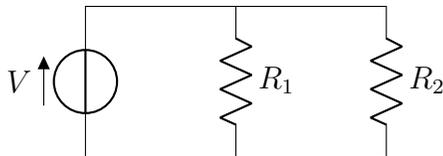
10. A point is moving on the circle

$$(x - 1)^2 + y^2 = 25.$$

At the point $(5, -3)$ the x -coordinate is increasing at the rate of 5 cm/sec . Find the rate of change of the y -coordinate at the same moment.

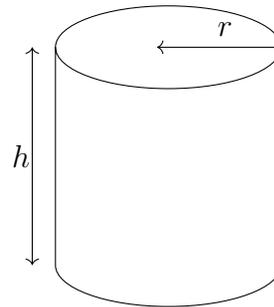
¹<https://pennance.us>

11. Water enters a cone of height 6 feet and radius 4 feet at a rate of 0.5 feet/sec. Find the rate of change of volume of water in the cone at the time τ when the water level is 2 feet. $\left(V = \frac{\pi r^2 h}{3}\right)$.
12. A balloon is launched from a point 50 m from an observer and rises vertically. If the rate of change of the *angle of elevation* θ is 0.1 rad s^{-1} , find the rate of change of the distance d between the observer and the balloon when $\theta = \pi/6$ radians.
13. A tank of water in the shape of a cone is leaking water at a constant rate of $2 \text{ m}^3 \text{ s}^{-1}$. The base radius of the tank is 5 m and the height of the tank is 14 m.
- (a) At what rate is the depth of the water in the tank changing when the depth of the water is 6 m?
- (b) At what rate is the radius of the top of the water in the tank changing when the depth of the water is 6 m?
14. When two resistors R_1 y R_2 are connected in parallel with a voltage source V , (see diagram),



the total resistance R , in ohms (Ω), satisfies $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If R_1 and R_2 are increasing at the rates of $0.4 \Omega \cdot \text{s}^{-1}$ and $0.9 \Omega \cdot \text{s}^{-1}$ respectively, how rapidly is R changing when $R_1 = 2 \Omega$ and $R_2 = 3 \Omega$.

15. The volume V of a spherical ball of radius R is increasing at a rate of $15 \text{ cm}^3 \text{ min}^{-1}$. Find:
- (a) The rate of change of the radius when $R = 60 \text{ cm}$.
- (b) The rate of change of radius **with respect to area** when $R = 60 \text{ cm}$.
16. An elastic cylinder



is compressed in such a way that its volume does not change. If the rate of change of the height h of the cylinder is 0.25 cm s^{-1} , find the rate of change of the radius when $r = 2 \text{ cm}$ and $h = 5 \text{ cm}$.