Math 3151 - Optimization with Constraints

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1. Find the dimensions of the rectangle of largest area with perimeter 500 cm.

2. Determine the area of the largest rectangle that can be inscribed in the unit circle.

3. A farmer purchases 1000 m of fencing in order to enclose a rectangular region bordering a straight river. Find the maximum area that can be surrounded if no fence is needed for the side adjacent to the river.

4. Find the point(s) on the parabola with equation $y = x^2 + 2$ which are closest to the point with coordinates $(0, 3)$.

5. Show that the maximum volume of a box with square base and total surface area $10\,\mathrm{m}^2$ is $\left(\sqrt{\frac{5}{3}}\right)^3\ \mathrm{m}^3$.

6. A rectangular poster of total area $400\,\mathrm{cm}^2$ must have top and bottom margins of 4 cm and side margins of 2 cm. Find the dimensions of the poster with largest **printed** area.

7. A rectangle has vertices at $(0, 0)$, $(x, 0)$ $(0, y)$ and at a point $P = (x, y)$ in the first quadrant which lies on the circle of radius 4 and center $(0, 0)$. Find:

   (a) The area of the rectangle as a function A of x only.

   (b) The domain I of the function A.

   (c) The critical points of A.

   (d) The absolute maximum value of A on the interval I.

8. A right triangle has its vertices at $O(0, 0)$, $A(12, 0)$, and $B(0, 20)$. Find the dimensions of the rectangle with largest area that can be inscribed in the triangle.

9. The entropy H of a 2-state system is given by: $H(p_1, p_2) = -\sum_{i=1}^{2} p_i \ln p_i$. Find the maximum of H subject to the constraint $p_1 + p_2 = 1$.

10. Find the point(s) on the graph of the relation $x = \sqrt{y}$ nearest the point (0, 4).

11. A cylinder with no top has surface area $100\,\mathrm{cm}^2$. Find the height h and base radius r which maximize the volume of the cylinder.

12. Find the maximum value of the product xy^2 if $x + y = 10$.

13. Show that the surface area of a cylinder of volume V is a minimum when the height is exactly twice the radius.

[1] https://pennance.us