Math 3151 -Exercises on the Mean Value Theorem

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1. Determine all numbers c that satisfy the conclusion of the mean value theorem for

$$f(x) = x^3 + 2x^2 + x$$

on the interval $[-2, 2]$.

2. Let $f(x) = px^2 + qx + r, p \neq 0$.

Find a point c (in terms of p, q, r, α, β on the interval $[\alpha, \beta]$ whose existence is guaranteed by the mean value theorem.

Answer: $c = \frac{\alpha+\beta}{2}$

3. Let $f(x) = x^3 - x, \quad x \in [-1, 2]$. Find (if they exist):

   (a) The intercept(s) of f on the horizontal axis.

   (b) The vertical intercept.

   (c) The interval(s) where f is increasing or decreasing and the extreme points of f.

   (d) The interval(s) where f is concave up or concave down and the inflection points of f.

4. Find the extreme values of the following functions in the given interval:

   (a) $f(x) = \dfrac{x}{4 + x^2}, \quad x \in [-3, 1]$

   (b) $f(x) = x^2 + 4x + 4, \quad x \in [-4, 1]$.

   (c) $f(x) = 2x^3 - 3x^2 - 12x + 2$, $x \in [-2, 3]$.

   (d) $f(x) = \sin x + \cos x$, $x \in [0, \pi]$.

   (e) $f(x) = \dfrac{1}{x^2 + 1}, \quad x \in \mathbb{R}$

   (f) $f(x) = (x-1)^2(x-3)^2$, $x \in [-1, 4]$

5. Let

$$f(x) = x + \cos x, \quad x \in [0, 2\pi].$$

Find (if they exist):

   (a) The roots of f.

   (b) The critical points of f.

   (c) The intervals upon which f is increasing/decreasing.

   (d) The intervals upon which f is concave up and concave down.

   (e) The inflection points of f.

   (f) Local and global extrema of f.

Sketch the graph of f.

6. The coordinate of a point undergoing rectilinear motion is given by

$$x(t) = t^3 - 12t^2 + 36t - 27, \quad 0 \leq t \leq 9.$$

Find:

   (a) The times at which the instantaneous velocity is zero.

   (b) The intervals upon which the point is speeding up. x(t).

   (c) The intervals upon which the point is moving in the negative direction.

   (d) The absolute maximum of the set

$$\{x(t) : t \in [0.9]\}$$

   and the time(s) at which this maximum is achieved.

   (e) The time(s) guaranteed by the mean value theorem at which the instantaneous velocity is equal to the average speed over the interval $[0, 1]$.

[1] https://pennance.us