

Math 3151 - Homework on Continuity

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1. Find the constants A and B such that the function

$$f(x) = \begin{cases} -A^2x + 5B, & \text{if } x \leq -5, \\ 20 - x, & \text{if } -5 < x < 10, \\ Bx - 10, & \text{if } x \geq 10. \end{cases}$$

is continuous for all real numbers.

2. Let $f(x) = \begin{cases} x^2 + 2, & \text{if } x \geq 1, \\ -kx + 1 & \text{if } x < 1. \end{cases}$

- (a) Find k if f is continuous at $x = 1$.
(b) Sketch the graph of f for the value of k found in part (a).
(c) Using the value of k found in part (a), determine if the function $g(x) = \frac{f(1+x) - f(1)}{x}$ has a limit at $x = 0$.

3. (a) State the Weierstrass Intermediate Value Theorem.

- (b) Use the Intermediate Value Theorem to approximate $\sqrt{2}$ with error less than 0.2.

4. Let $f(x) = \begin{cases} x^2, & \text{if } x < 1, \\ 3 - x & \text{if } x \geq 1. \end{cases}$ Is f continuous at $x = 1$? Use limits to justify your answer. If f is discontinuous at $x = 1$, indicate the type of discontinuity.

5. Let $f(x) = \begin{cases} x^2 + 1, & \text{if } x < 2, \\ 8 - x, & \text{if } x \geq 2. \end{cases}$ Is f continuous at $x = 2$? Use limits to justify your answer. If f is discontinuous

at $x = 1$, indicate the type of discontinuity (removable, jump or infinite).

6. Let

$$f(x) = \begin{cases} \sqrt{3x-4}, & \text{if } 4/3 \leq x < 2 \\ -x^2 + 6, & \text{if } 2 < x < 3 \\ -\frac{15}{x+2} & \text{if } 3 < x \leq 4 \\ -\frac{13-7x}{4x-10} & \text{if } 4 < x \leq 5 \\ \frac{1}{(x-5)^2} & \text{if } 5 < x < 6 \\ -2, & \text{if } x = 6 \\ \cos(x-6), & \text{if } x > 6. \end{cases}$$

Locate the points at which f has a:

- (a) removable discontinuity.
(b) jump discontinuity.
(c) infinite discontinuity.

7. Explain how $\sqrt[3]{7}$ could be approximated using the Intermediate Value Theorem.

8. Show that

$$\tan x = 1 - x$$

has a root in $(0, \pi/2)$.

9. Let $f(x) = x^3 - 4x + 1$. Show that the equation $f(x) = 0$ has a solution in the closed interval $[0, 1]$.

10. Let f be a continuous bijection. If $\lim_{x \rightarrow a} f(x) = b$ show that $\lim_{y \rightarrow b} f^{-1}(y) = a$.

¹<https://pennance.us>