

UNIVERSITY OF PUERTO RICO – RÍO PIEDRAS
DEPARTMENT OF MATHEMATICS

REVIEW FOR CALCULUS I – EXAM IV
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1. Find the following limits:

$$(a) \lim_{x \rightarrow 0} \frac{\sin 5x}{x} \qquad (c) \lim_{x \rightarrow 5} \frac{3x + 1}{x^2 + 8} \qquad (e) \lim_{x \rightarrow 0} (\cot x - \csc x)$$

$$(b) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \qquad (d) \lim_{x \rightarrow 0} \frac{3x}{\sin 11x} \qquad (f) \lim_{x \rightarrow \infty} \frac{4x^2 - 3x + 5}{x^2 + 2x + 3}$$

2. Evaluate the following derivatives

$$(a) \frac{d}{dx} \left[\frac{e^{x^2}}{x^4 + 1} \right] \qquad (d) \frac{d}{dx} [\ln(\ln x)] \qquad \frac{d^3}{dx^3} (e^x \sin x)$$
$$(b) \frac{d}{dx} [\tan(\ln x)] \qquad (e) \frac{d}{dt} \left[\frac{e^t}{t^2 + t} - \frac{1}{\sqrt{t}} \right] \qquad \frac{d^2}{dx^2} \left[(1 + 2x) \frac{d}{dx} (5 - x^3) \right]$$
$$(c) \frac{d}{dt} \left[2te^t - \frac{1}{\sqrt{t}} \right] \qquad (f) \frac{d}{dx} [\ln(\ln x)] \qquad \frac{d^2}{dx^2} (e^{2x} \cos x)$$

3. Suppose that L and E are differentiable functions which satisfy

$$xL'(x) = 1 \text{ and } L(E(x)) = x$$

Show that $E'(x) = E(x)$.

4. Use implicit differentiation to find the derivative $\frac{dy}{dx}$ and the equation of the tangent to the curve $2x^3 + 2y^3 = 9xy$ at the point $(2, 1)$.

5. Find the equation of the tangent and normal line to the curve $x^2 - 4y^2 = 9$ at the point $(5, 2)$.

6. Let $g(x) = 3x^4 - 6x^2 + 5$. Determine:

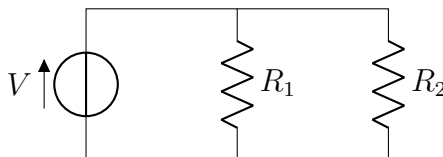
- (a) the intervals in which g is increasing.
- (b) the intervals in which g is concave up.
- (c) the points of inflection of the graph (if any).

Sketch the graph of g .

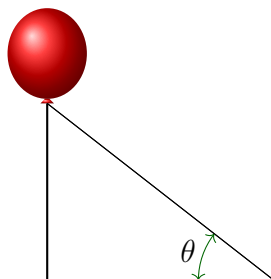
7. Find the equation of the tangent and normal line to the curve $x = \cos y$ at the point $(\frac{1}{2}, \frac{7\pi}{3})$.

8. Find the equation of the tangent and normal line to the curve $x^2 - 4y^2 = 9$ at the point $(5, 2)$.

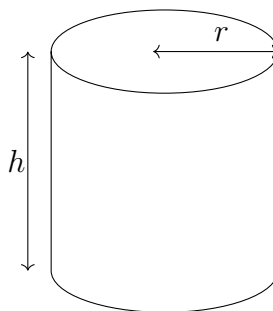
9. When two resistors R_1 y R_2 are connected in parallel with a voltage source V , (see diagram),



- the total resistance R , in ohms (Ω), satisfies $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If R_1 and R_2 are increasing at the rates of $0.4 \Omega \cdot \text{s}^{-1}$ and $0.9 \Omega \cdot \text{s}^{-1}$ respectively, how rapidly is R changing when $R_1 = 2 \Omega$ and $R_2 = 3 \Omega$.
10. A balloon is launched from a point 50 m from an observer and rises vertically. If the rate of change of the *angle of elevation* θ is 0.1 rad s^{-1} , find the rate of change of the distance d between the observer and the balloon when $\theta = \pi/6$ radians .



11. A rectangle is changing in such a way that its area does not change. If the area is 200 m^2 and the rate of change of the longest side is $4 \text{ m} \cdot \text{s}^{-1}$, find the length of the smaller side radius when its rate of change is $0.5 \text{ m} \cdot \text{s}^{-1}$.
12. An elastic cylinder



is compressed in such a way that its volume does not change. If the rate of change of the height h of the cylinder is $0.25 \text{ cm} \cdot \text{s}^{-1}$, find the rate of change of the radius when r is 2 cm and $h = 5 \text{ cm}$.

13. The coordinate of a point undergoing rectilinear motion is given by

$$x(t) = -t^2 + 4t + 5.$$

Find:

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| (a) The time τ at which the instantaneous velocity is zero. | (d) The maximum element of the set $\{x(t) : t \in \mathbb{R}\}$. |
| (b) The set $\{t \in \mathbb{R} : x(t) \text{ is speeding up}\}$. | (e) The time t_0 guaranteed by the mean value theorem at which the instantaneous velocity $v(t_0)$ is equal to the average speed over the interval $[0, 1]$. |
| (c) The time or times at which $x(t)$ is a maximum. | |

14. Find the following antiderivatives:

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| (a) $\int x^{-2} + \frac{1}{\sqrt{x}} dx.$ | (e) $\int x^2 + \sqrt{x} + 3x^{-2} dx$ |
| (b) $\int x^3 e^{x^4} dx.$ | (f) $\int \frac{x^3}{x^2 + 1} dx$ |
| (c) $\int \sin(3x - 1) dx.$ | (g) $\int x \sin x dx$ |
| (d) $\int \frac{\cos x}{\sqrt{1 + \sin x}} dx.$ | |

15. Let f be a function which is continuous and increasing in the interval $[0, 1]$. Let $P = \{x_0, x_1, \dots, x_n\}$, $n \in \mathbb{N}$ be a partition of the interval such that $x_i - x_{i-1} = \frac{1}{n}$, $i = 1, 2, \dots, n$. Let $L(f, P)$ and $U(f, P)$ be the lower and upper sums of f for the partition P . Verify that $U(f, P) - L(f, P) = \frac{f(1) - f(0)}{n}$.

16. Evaluate the following definite integrals:

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|---|--|
| (a) $\int_0^{\pi/4} \tan^3(x) \sec^2(x) dx$ | (c) $\int_{\pi/6}^{\pi/2} \sin 2x \cdot \cos^3 2x dx.$ |
| (b) $\int_{-1}^1 \frac{e^x}{e^x + 5} dx.$ | |

17. If

$$\int_1^5 f(x) dx = 4, \quad \int_5^7 f(x) dx = 3, \quad \int_1^7 g(x) dx = 5, \quad \int_5^7 g(x) dx = 6$$

find:

$$(a) \int_1^5 f(x) + 2g(x) dx.$$

$$(b) \int_7^5 2f(x) - 3g(x) dx.$$

18. If $\int_2^5 f(x) dx = 4$, and $\int_4^2 f(x) dx = 3$, find: $\int_5^4 2f(x) dx$.

19. Evaluate $\frac{d}{dx} \left(\int_1^{\ln x} \cos t dt \right)$.

20. If $x'(2) = 4$ and $x'(0) = 0$ find $\int_0^2 x''(t)x'(t) dt$.

21. Suppose that $f(x)$ is continuous for all $x \in \mathbb{R}$ and that $\int_0^x f(t) dt = \sin x - x \cos x$.
Find $f(\frac{\pi}{6})$

22. Find the area of the region Ω contained in the strip $0 \leq x \leq \pi$ and lying between the curves $y = \cos x$ and $y = \frac{1}{2}$.

23. Find the area of the region between the curves $y = x^2$ and $y = x^4$ where $0 \leq x \leq 1$.

24. Find the area of the region Ω lying between the curves $y = \sin x$; $y = \cos x$,
from $x = \pi/4$ to $x = 5\pi/4$.

25. Find the mean value of the function $f(x) = \cos x$; $0 \leq x \leq \pi$

26. Suppose that $\int_a^b g(x) dx \neq 0$. Let $\lambda(x) = \frac{g(x)}{\int_a^b g(x) dx}$. Simplify $\int_a^b \lambda(x) dx$.

27. (a) Define the number π .

(b) Use the definition in part (a) to show that the area of a circle of radius r is πr^2 .

28. Suppose that f and its derivative f' are differentiable functions such that

$$f''(x) = 3x^3 - 4x^2, \quad f'(1) = 4 \text{ and } f(1) = 2.$$

Find $f(x)$.

29. During a chemical reaction $A \longrightarrow mM + nN$, the concentration $[A]_t$ of reactant A at time t satisfies the first order reaction equation $\frac{d[A]}{dt} = -3[A]$. Let $[A]_0$ denote the initial concentration. Derive an expression for the time τ at which $[A]_\tau = \frac{1}{2}[A]_0$