MATE 3151 -Review for Exam III

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1. Compute the following definite integrals.

   (a) $\displaystyle\int_0^1 \frac{\sqrt{x}}{(1+x^{3/2})^9}dx$

   (b) $\displaystyle\int_0^{\pi/4} \tan^5(x)\sec^2(x)dx$

   (c) $\displaystyle\int_4^2 \frac{2e^x+2}{\sqrt{e^x+x+5}}dx$

   (d) $\displaystyle\int_0^{\pi^4} \frac{\ln(t+\pi^4)}{t+\pi^4}dt$

   (e) $\displaystyle\int_0^2 u^3(u^2+1)^6\,du$ (Hint: put $z=u^2+1$)

   (f) $\displaystyle\int_0^1 (x-2)(x-3)\,dx$

   (g) $\displaystyle\int_{-1}^1 \frac{e^x}{e^x+5}dx.$

   (h) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x\cdot\cos^3 2x dx.$

   (i) $\displaystyle\int_0^1 12e^{3x-2}\,dx$

   (j) $\displaystyle\int_4^7 1-\sin(\pi x)\,dx$

   (k) $\displaystyle\int_0^1 \frac{48x^3+32x}{(3x^4+4x^2+9)^{5/2}}\,dx$

   (l) $\displaystyle\int_0^7 \frac{\ln(x+1)}{x+1}\,dx$

   (m) $\displaystyle\int_{\ln\frac{\pi}{6}}^{\ln\frac{\pi}{3}} e^{2x}\cos(e^{2x})\,dx$

2. Evaluate:

   (a) $\displaystyle\int_0^3 \left[\frac{d}{dx}\left(\sqrt{7+x^2}\right)\right]\,dx.$

   (b) $\displaystyle\frac{d}{dx}\left(\int_6^{1+x^2}\frac{dt}{\sqrt{3t+5}}\right).$

   (c) $\displaystyle\int_0^2 \left[\frac{d}{dx}\left(1+x^2\right)^3\right]\,dx.$

   (d) $\displaystyle\frac{d}{dx}\left(\int_2^{x^3}\frac{\sin t}{e^t+1}\,dt\right).$

   (e) $\displaystyle\frac{d}{dx}\left(\int_2^{x^3}\frac{\ln t}{e^t}\,dt\right).$

3. Use symmetry considerations to evaluate the following integrals:

   (a) $\displaystyle\int_{-2}^2 x^3-x\,dx$

   (b) $\displaystyle\int_{-\pi}^{\pi} \frac{\sin x^{13}}{1+x^2}dx.$

   (c) $\displaystyle\int_{-\pi}^{\pi} \sin x^3 dx.$

   (d) $\displaystyle\int_{-\pi}^{\pi} |x|\,dx.$

4. Let $f(x)=2x$, $x\in\mathbb{R}$. Let $\mathcal{P}$ be the uniform partition of the interval $[1,5]$ into **four** subintervals.

   (a) List the points of the partition $\mathcal{P}$.

   (b) Evaluate the lower sum $L(f,\mathcal{P})$.

   (c) Evaluate the upper sum $U(f,\mathcal{P})$.

   (d) Compute $\displaystyle\int_1^5 f(x)dx$ using the definition.

[1] https://pennance.us

5. Let $f(x) = x + 1, \quad 0 \le x \le 1$. Let $P_n = \{x_0, x_1, \ldots, x_n\}$ be a partition of the interval $[0, 1]$ such that

$$x_i - x_{i-1} = \frac{1}{n}, \quad i = 1, 2, ..., n.$$

   (a) Give a definition of the concept of *definite integral.*

   (b) Find the lower sum $L(f, P_n)$.

   (c) Find the upper sum $U(f, P_n)$.

   (d) Find the limit of the lower sum as $n \to \infty$.

   (e) Show that that

$$U(f, P) - L(f, P)) = \frac{f(1) - f(0)}{n}.$$

6. (a) Sketch the graph of the function $f(x) = \begin{cases} x^2 + x, & 0 \le x \le 1 \\ 2x, & 1 < x \le 3. \end{cases}$

   (b) Is f continuous at $x = 1$?

   (c) Find the function

$$F(x) = \int_0^x f(t)\, dt, \quad 0 \le x \le 3,$$

   and sketch its graph.

7. Compute the area of the region Ω bounded by the $x-$axis the line $x + y = 0$ and the line $5x - 6y = 120$.

8. Let Ω be the region bounded by the graphs of the functions $y = x^2$ and $y = 4 - x^2$. Sketch the region Ω and calculate the area A of Ω.

9. Use integration to find the area of the triangle with vertices $A(0, 8)$, $B(8, 8)$, $C(4, 10)$. Verify your answer using the usual formula for the area of a triangle.

10. Let $f(x) = 12(x + 1)^5$.

    (a) Determine the average value of $f(x)$ on the interval $[0, 2]$.

    (b) Find all numbers c in the interval $(0, 2)$ which satisfy the conclusion of the Mean Value Theorem for Integrals for the function f.

11. An object moves along a line so that its velocity at time t is

$$v(t) = -t^3 + 3t^2 - 2t.$$

    (a) Find the displacement $v(3) - v(0)$ of the object during the time interval $0 \le t \le 3$.

    (b) Find the time interval on which the velocity $v(t)$ is positive.

    (c) Calculate the total distance traveled during the time interval $0 \le t \le 3$.

12. An object starts at the origin and moves along the x-axis with velocity

$$v(t) = 10t - t^2, \quad 0 \le t \le 10.$$

    (a) What is the position of the object at any time t, $0 \le t \le 10$?

(b) When is the object's velocity a maximum, and what is its position at that time?

13. Suppose x and y are related by $x = \int_{12}^{y} \frac{1}{\sqrt{1+4t^2}} dt$. Prove that $\frac{d^2y}{dx^2}$ is proportional to y and find the constant of proportionality.

14. Suppose that $\int_1^8 f(x)\,dx = 12$. Find $\int_1^2 x^2 f(x^3)\,dx$.

15. Solve the differential equation $\frac{dx}{dt} = -3t$ with initial condition $x(0) = 1$.

16. Solve the differential equation $\frac{dx}{dt} = -3x$ with initial condition $x(0) = 1$.

17. Let f be a continuous function. Show that

$$\int_{a+c}^{b+c} f(x-c)dx = \int_a^b f(x)dx.$$

18. Supppose that the function $x(t)$ is twice differentiable on the interval $[0, 2]$. If $x'(2) = 4$ and $x'(0) = 0$ find $\int_0^2 x''(t)x'(t)\,dt$.

19. Find $x(t)$ if $x''(t) = -1$ and $x'(0) = 1$, and $x(0) = 0$.

20. Find the function f if $\int_2^x e^{-t} f(t)\,dt = (2-5x^2)^5$.

21. Suppose that $\int_1^8 f(x)\,dx = 12$. Find $\int_1^2 x^2 f(x^3)\,dx$.

22. If

$$\int_1^5 f(x)\,dx = 3, \quad \int_5^7 f(x)\,dx = 4, \quad \int_1^7 g(x)\,dx = 4, \quad \int_5^7 g(x)\,dx = 5$$

find:

(a) $\int_1^5 3f(x) - 2g(x)\,dx$ (b) $\int_7^5 2f(x) + 4g(x)\,dx$

23. Let f be odd. Let F be any antiderivative of f. Show that:

(a) f' is even. (b) F is even.

24. Let f be a continuous function. if $\int_0^x tf(t)dt = \frac{2x}{4+x^2}$.

(a) Determine $f(0)$. (b) Solve $f(x) = 0$.