MATE 3151 -Review for Exam II

Philip Pennance[1] –Semester I, 2023-24

1. Find f':

   (a) $f(x) = \dfrac{x^2-1}{x^2+1}$

   (b) $f(x) = \sqrt[3]{x^2+3x}$

   (c) $f(x) = \operatorname{sen}^3(3x-1)$

   (d) $f(x) = \left(\dfrac{e^x+3}{3-e^x}\right)^3$

2. Evaluate:

   (a) $\dfrac{d}{dx}\dfrac{e^x \ln x}{\cos x}$

   (b) $\dfrac{d}{dx}\dfrac{\sqrt{x+13}-4}{x-3}$

   (c) $\dfrac{d}{dx}\left[\sin(3\pi - x)\ln(x^3+1)\right]$

   (d) $\dfrac{d}{dx}\left[\tan^3(\sin(\pi x - 3))\right]$

3. Find the following antiderivatives:

   (a) $\displaystyle\int x^4 + 2x^2 - x + 1\,dx$

   (b) $\displaystyle\int \frac{1}{x} + \frac{2}{3x+4}\,dx$

   (c) $\displaystyle\int x\sqrt{x} + x^{3/2}\,dx$

   (d) $\displaystyle\int \tan(3x+4)\,dx$

   (e) $\displaystyle\int \cos\theta e^{\sin\theta}\,d\theta$

   (f) $\displaystyle\int \mathrm{id}[x(t)]x'(t)\,dt$

   (g) $\displaystyle\int x''(t)\,dt$

   (h) $\displaystyle\int xe^{x^2}\,dx$

   (i) $\displaystyle\int \frac{1}{x}\,dx$

   (j) $\displaystyle\int \ln x\,dx$

   (k) $\displaystyle\int \frac{2x+4}{x^2+4x+5}\,dx$

   (l) $\displaystyle\int \cot t\,dt$

   (m) $\displaystyle\int 2^x\,dx$

4. Show that the slope m of the curve $x = \sin y$ at the point (x,y) is given by
$$m = \sec y.$$

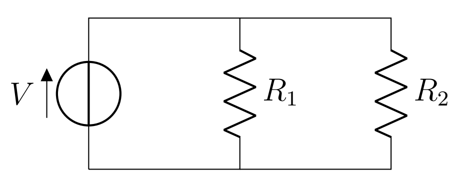
5. Suppose that $y = f(x)$. If
$$2xy + \pi\sin y = 2\pi.$$
   Find:

   (a) An expression for y' in terms of x, y.

   (b) An expression for y'' in terms of x, y.

   (c) An equation for the tangent line to the graph of f at the point $(1, \frac{\pi}{2})$.

6. When two resistors R_1 y R_2 are connected in parallel with a voltage source V, (see diagram), the total resistance R, in ohms (Ω), satisfies
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



[1] https://pennance.us

If R_1 and R_2 are increasing at the rates of $0.4\,\Omega\cdot\mathrm{s}^{-1}$ and $0.9\,\Omega\cdot\mathrm{s}^{-1}$ respectively, how rapidly is R changing when $R_1 = 2\,\Omega$ and $R_2 = 3\,\Omega$.

7. A spherical ballon is expanding in such a way that its volume is increasing at 6 cubic centimetres per second. Find the rate of increase of the surface of the ballon when its volume is $36\pi\,\mathrm{cm}^3$.

8. A 20 ft ladder is resting against the wall with its bottom 10 ft from the wall. The base of the ladder is pushed towards the wall at a rate of 0.5 ft/sec, how fast is the top of the ladder moving up the wall 5 seconds later?

9. A right triangle has its vertices at $O(0,0)$, $A(12,0)$, and $B(0,20)$. Find the dimensions of the rectangle with largest area that can be inscribed in the triangle.

10. For each of the following functions, find the critical points, the intervals upon which the function is increasing, local extreme values, the concavity of the graph, and inflection points (if any).

    (a) $f(x) = (x-2)^4 - 2(x-2)^2$.

    (b) $f(x) = xe^{x^2-3x}$.

11. Give an example of a function defined the set $(0,\infty)$ which is increasing and concave down and unbounded above. Verify that your example has all the required properties.

12. Find the extreme values of the following functions in the given interval:

    (a) $f(x) = x^2 + 4x + 4, \quad x \in [-4, 1]$.

    (b) $f(x) = 2x^3 - 3x^2 - 12x + 2$, $x \in [-2, 3]$.

    (c) $f(x) = \sin(x) + \cos(x)$, $x \in [0, \pi]$.

    (d) $f(x) = \dfrac{1}{x^2+1}, \quad x \in \mathbb{R}$

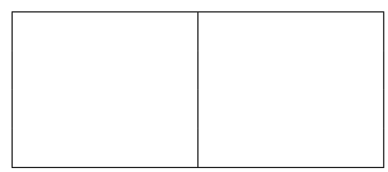
    (e) $f(x) = (x-1)^2(x-3)^2$, $x \in [-1, 4]$

13. Suppose that at time $t = 0$ a life guard is located at point $(0,0)$ in the (x,y)-plane. A swimmer is drowning at point (a,b) in the first quadrant of the same coordinate system. If speed of the life guard is

$$v(x) = \begin{cases} v_1 & \text{if } 0 \le x \le c, \\ v_2 & \text{if } c < x \le a. \end{cases}$$

    How can the lifeguard rescue the swimmer in minimum time? Justify your answer.

14. A rectangular garden is divided into two equal parts as shown. Find the dimensions of the garden which maximize the area if the total length of fence is 1200 m.



15. (a) State the Intermediate Value Theorem.

    (b) Indicate briefly how this theorem could be used to estimate $\sqrt{3}$.

16. Find the number $c \in (0,2)$ guaranteed by the mean value theorem for the function

$$f(x) = \frac{x}{x+1}.$$

17. The coordinate of a point undergoing rectilinear motion is given by

$$x(t) = -t^2 + 4t + 5.$$

    Find:

    (a) The velocity and acceleration of the point.

    (b) The time τ at which the instantaneous velocity is zero.

(c) The set

$$\{t \in \mathbb{R} : x(t) \text{ is speeding up}\}.$$

(d) The maximum of the set

$$\{x(t) : t \in \mathbb{R}\}.$$

(e) The set $\{t \in \mathbb{R} : x(t) = 0\}$.

(f) The time t_0 guaranteed by the mean value theorem at which the instantaneous velocity $v(t_0)$ is equal to the average speed over the interval $[2, 10]$.

18. A point of mass m moving along a line experiences a force F which is a function of its position. Suppose that x satisfies the equation of motion:

$$m\frac{d^2x}{dt^2} = F(x(t)). \tag{1}$$

Suppose also that there exists a function of position V such that for all x,

$$F(x) = -V'(x).$$

Show that if $x(t)$ is a solution of (1), then quantity

$$E = \frac{m}{2}\dot{x}(t)^2 + V(x(t)) \tag{2}$$

is a constant of the motion.

19. A man of height 6 ft is walking with speed 3 ft/sec away from a lampost of height 18 ft. Find the rate of change of the length of his shadow when he is 10 ft from the post.

20. Show that if temperature of the earth is continuous, then their exist two antipodal points on the surface which have the same temperature.