MATE 3151 -Review for Exam I

Philip Pennance[1] –Semester I, 2023-24

Prove using the (ϵ, δ) definition:

1. $\lim_{x\to 0} x = 0$
2. $\lim_{x\to 2} (3x-1) = 5$
3. $\lim_{x\to 4} \sqrt{x} = 2$

Evaluate:

1. $\lim_{x\to -1} \dfrac{x^{13}+1}{x+1}$
2. $\lim_{x\to 64} \dfrac{x^{2/3}-16}{\sqrt{x}-8}$
3. $\lim_{x\to 0} \tan(3x)\csc(5x)$
4. $\lim_{x\to -3} \dfrac{x^3-4x+15}{x^2-x-12}$
5. $\lim_{x\to 1} \dfrac{2x^2-x-1}{x^2-1}$
6. $\lim_{x\to 0} \dfrac{x^2}{1-\cos 2x}$
7. $\lim_{x\to 2^+} \dfrac{\sqrt{2x}-2}{\sqrt{x-2}}$
8. $\lim_{x\to 0} \dfrac{\sqrt{4-x}-2}{x}$
9. $\lim_{x\to \infty} \dfrac{\sin\frac{1}{x^2}}{\frac{1}{x^2}}$
10. $\lim_{x\to \infty} \dfrac{x^{3/2}}{x^{5/2}+2x-1}$
11. $\lim_{x\to \infty} \dfrac{\frac{x^3}{(1+x^3)^2}}{\frac{1}{x^3}}$
12. $\lim_{x\to \infty} \dfrac{\sin^2(\frac{1}{\sqrt{x}})}{\frac{1}{x}}$
13. $\lim_{x\to \infty} \cos\left(\dfrac{1}{x^2}\right)$

---

1. Suppose that $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = 0$. If $L \neq 0$ show that $\dfrac{f}{g}$ has no limit at the point $x = c$.
2. Let $f(x) = \begin{cases} x^2+2, & \text{if } x \geq 1, \\ -kx+1 & \text{if } x < 1 \ . \end{cases}$
   (a) Find k if f is continuous at $x = 1$.
   (b) Sketch the graph of f for the value of k found in part (a).
   (c) Determine if f is differentiable at $x = 1$. Justify your answer.
3. Let $f(x) = \begin{cases} ax+b & \text{if } x \leq -1, \\ ax^3+x+2b, & \text{if } x > -1 \end{cases}$
   (a) Find b if f is continuous over $\mathbb{R}$.
   (b) Find a and b if f is differentiable over $\mathbb{R}$.
4. Use the definition of the derivative to show that $f(x) = x^2 - x - 1$ is differentiable at $x = 1$.
5. Let $f(x) = (x-2)(x^2-x-11)$. Find all points at which the tangent line to the graph of f is horizontal.
6. Find the equation of the tangent and normal line to the curve $x^2 - 4y^2 = 9$ at the point $(5, 2)$.

[1] https://pennance.us

7. Find f':

   (a) $f(x) = \dfrac{x^2-1}{x^2+1}$

   (b) $f(x) = \sqrt[3]{x^2+3x}$

   (c) $f(x) = \text{sen}^3(3x-1)$

   (d) $f(x) = \left[\left(\dfrac{e^x+3}{3-e^x}\right)^3\right]$

8. Evaluate:

   (a) $\dfrac{d}{dx}\dfrac{e^x \ln x}{\cos x}$

   (b) $\dfrac{d}{dx}\left[\dfrac{\sqrt{x+13}-4}{x-3}\right]$

   (c) $\dfrac{d}{dx}\left[\sin(3\pi - x)\ln(x^3+1)\right]$

   (d) $\dfrac{d}{dx}\left[\tan^3\left(\sin(\pi x - 3)\right)\right]$

9. Differentiate:

   (a) $f(x) = 2x\sin(3x)$

   (b) $r(\theta) = \sin(3\theta - \pi)^2$

   (c) $\rho(\theta) = \sin^2(3\theta - \pi)$

   (d) $g(\theta) = \sqrt{a^2 - \sin^2\theta}$

   (e) $\beta(t) = \dfrac{1}{\sqrt{2-\cos t}}$

   (f) $f(x) = \sin(\sin x)$

10. Find the second derivative:

    (a) $g(x) = x^6$

    (b) $s(t) = \sqrt{1+t}$

    (c) $x(t) = \dfrac{t^2+5t+2}{t+3}$

    (d) $x(t) = \dfrac{t^2+2}{t^3+1}$

    (e) $x(t) = \sqrt[4]{3-2t^2}$

    (f) $f(z) = \dfrac{z^2+1}{z}$

11. Find the derivative of:

    (a) $f(x) = e^{-(x-1)^2}$

    (b) $h(x) = xe^{\tan x}$

    (c) $f(x) = e^{-2x}\sin x$

    (d) $g(t) = e^{(1+3t)^2}$

    (e) $f(x) = \sqrt{10^{5-x}}$

    (f) $f(t) = 2te^t - \dfrac{1}{\sqrt{t}}$

    (g) $f(z) = \dfrac{1}{(e^z+1)^2}$

    (h) $f(\theta) = \dfrac{1}{1+e^{-\theta}}$

    (i) $h(t) = \ln\left(e^{-t} - t\right)$

12. Show that if f and g are differentiable, then so is $f+g$ and that

$$(f+g)'(x) = f'(x) + g'(x)$$

13. The position x of a point on in rectilinear motion is given as a function of time by

$$x(t) = \frac{1}{t+2}, \quad t \geq 0.$$

    Find the position, velocity and acceleration when $t = 0$.

14. Calculate $\dfrac{d^2}{dx^2}\left[(1+2x)\ \dfrac{d}{dx}\ (5-x^3)\right]$