Department of Mathematics

Math 3151-Calculus I (Section 6)
Semester II, 2023-24

Time: LW 2:30 pm - 3:20 pm and MJ 2:30 pm - 3:50 pm
Room: C-212
Instructor: Philip Pennance
C-118, ext-88264
Email: philip.pennance1@upr.edu
Office hours: M W 10:30 AM–1:00 PM (or by appointment).
Course page: https://online.uprrp.edu/course/view.php?id=76700

Course Description

MATH 3151. Calculus I. *Four credits. Five hours of lecture per week. Prerequisite: 4 points or more in Advanced Placement II of the College Board or equivalent*

Limits and continuity of functions of one variable, differentiation and its application to optimization. Antiderivatives. The mean value theorem, corollaries. The definite integral and its applications. Definition and properties of the logarithmic and exponential functions using the calculus.

Course Objectives

After this course the students are expected to:

1. be competent in the methods of mathematical reasoning and proof relating to each part of the syllabus.
2. have developed their skills in thinking logically, formulating precise mathematical arguments, solving problems and presenting solutions in a good mathematical style.
3. have become familiar with the definitions, concepts, results and methods of proof relating to each part of the syllabus;
4. be able to quote the definitions and results, and to reproduce the proofs of some key results.
5. be able to solve problems relating to the material covered. These might be straightforward applications of the definitions and results but might also be problems of a more testing nature.

Contenido Temático

Bosquejo del contenido y Distribución del Tiempo

Lección	Sección	Tópicos	Asignación
1–2	1.1, 1.2, 1.3	Repaso de algunos conceptos de Precálculo	
3–4	2.1	Tasas de cambio y tangentes a curvas	1–11, 17–20
5–7	2.2	Límite de una función y leyes de límites	1–6, 11–80
8–9	2.3	La definición formal del límite	15–50, 56
10–11	2.4	Límites laterales	1–40
12–13	2.5	Continuidad	1–46, 51–63
14–15	2.6	Límites que envuelven el infinito; asíntotas de gráficas	1–62
16	3.1	Rectas tangentes y la derivada en un punto	5–36
17–18	3.2	La derivada como función	1–32, 37–58
		Primera evaluación del profesor P_1	
19–21	3.3	Reglas de diferenciación	1–42, 45–65
22–23	3.4	La derivada como tasa de cambio	1–25
24–25	3.5	Derivadas de las funciones trigonométricas	1–44, 55–62
26–27	3.6	La regla de la cadena	1–76
28–29	1.6, 3.8	Funciones inversas y sus derivadas	1–44
30	1.5, 1.6, 7.2	Logaritmos naturales (derivadas solamente)	5–75
31	1.5, 1.6, 7.3	Funciones exponenciales (derivadas solamente)	5–119
32–33	3.7	Diferenciación implícita	1–49
34–35	3.10	Tasas relacionadas	1–44
		Evaluación departamental de medio término D_1	
36–37	4.1	Valores extremos de funciones	1–76
38–40	4.2	El teorema de la media (o valor medio)	1–54, 58–64
41–42	4.3	Funciones monotónicas y el criterio de la primera derivada	1–52
43–44	4.4	Concavidad y trazado de curvas	1–92
45–47	4.6	Problemas de optimización	1–15, 18–39
48	4.7	El método de Newton	1–12, 19–23
49–50	4.5	Formas indeterminadas y la regla de L'Hôpital	1–83
51–53	4.8	Antiderivadas	1–64
54	5.1	Área y su estimación mediante sumas finitas	1–18
55–56	5.2	Notación sigma y límites de sumas finitas	1–46
57–58	5.3	La integral definida	1–80
59–61	5.4	El teorema fundamental del cálculo	1–62
62–64	5.5	Integrales indefinidas y el método de sustitución	1–63
65–67	5.6	Sustitución y área entre curvas	1–62
		Segunda evaluación del profesor P_2	
68–69	6.1	Volumen de sólidos: capas, discos y arandelas	15–50
70–71	3.8, 7.1	Logaritmos naturales (en detalle)	5–75
72–73	3.8, 7.1	Funciones exponenciales (en detalle); aplicación al cambio exponencial	5–119
74–75	7.2	Cambio exponencial y ecuaciones diferenciales separables	1–46
		Evaluación departamental final D_2	

Week	Link to notes and additional exercises in course page	Exercises
1	Review of Precalculus	
2	Limits of sequences	Exercises
2	Limits of funcions	Exercises
3	Continuity	Exercises
4	The derivative	Exercises
5	The derivative, Part II	Exercises
EXAM I		Practice
6	Implicit differentiation, related rates	Exercises
7	Logarithms	See notes
8	The Mean Value Theorem (MVT) and consequences	Exercises
9	Optimization	Exercises
EXAM II		Practice
10	Antiderivatives	Exercises
11	Antiderivatives Part II	
12	The definite Integral	
13	The Fundamental Theorem of Calculus (FTC)	Exercises
	Area under a curve	Exercises
	Volumes	Exercises
EXAM III		Practice
14	Logarithms in detail. Exponential functions	Exercises
	Exponential growth	Exercises
15	Applications	Exercises
EXAM IV		

Syllabus with Notes and Selected Formulae

Review of Precalculus (1 week)

1. Review of Binary Relations and Functions
2. Sequences.
3. The Limit of a Sequence.

Limits (1 Week)

4. The Limit of a Function.

   Let a be a point in an open interval (c, d) and f a function defined at every point of (c, d) except possibly at a. Then f has a *limit* at a if there exists a number L with the property that $\forall \epsilon > 0$, $\exists \delta > 0$ such that $\forall x$,

   $$0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$$

5. The number L in the above definition is unique and is called the *limit* of f at a and is denoted $\lim_{x\to a} f(x)$.

6. The following are equivalent:

   (a) $\lim_{x\to a} f(x) = L$.

   (b) $\lim_{x\to 0} f(x + a) = L$.

7. The following are equivalent:

   (a) $\lim_{x\to a} f(x) = L$.

   (b) $\lim_{x\to a} f(x) - L = 0$

8. The following are equivalent:

   (a) $\lim_{x\to a} f(x) = 0$

   (b) $\lim_{x\to a} |f(x)| = 0$

9. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then

(a) $\lim_{x \to a} f(x) + g(x) = L + M$

(b) $\lim_{x \to a} \alpha f(x) = \alpha L$ for all $\alpha \in \mathbb{R}$.

(c) $\lim_{x \to a} f(x)g(x) = LM$.

(d) If $\lim_{x \to a} g(x) = M$ and $M \neq 0$, then $\lim_{x \to a} \frac{1}{g(x)} = \frac{1}{M}$.

(e) If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, and $M \neq 0$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}.$$

(f) Let P and Q be polynomials with $Q(a) \neq 0$, then

$$\lim_{x \to a} \frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)}.$$

Continuity (1 Week)

10. Continuity.

Let a be a point in an open interval (c, d) and f a function defined at every point of (c, d) (including a). The function f is *continuous* at a if

$$\lim_{x \to a} f(x) = f(a).$$

Equivalently, f is *continuous* at a if

$$\lim_{h \to 0} f(a + h) = f(a).$$

11. Boundedness Theorem

A continuous function f in the closed interval $[a, b]$ is bounded on that interval. That is, there exist real numbers m and M such that:

$$m < f(x) < M \quad \text{for all } x \in [a, b].$$

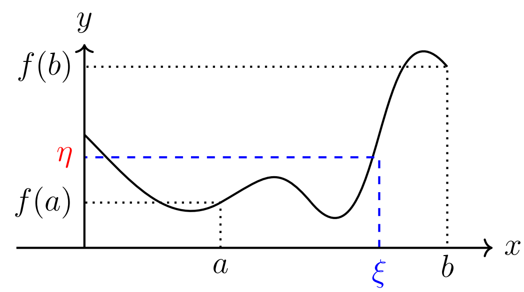
12. Extreme Value Theorem.

If f is continuous in the closed and bounded interval $[a, b]$, then f has a maximum and a minimum. I.e., there exist numbers c and d in $[a, b]$ such that:

$$f(c) \leq f(x) \leq f(d) \quad \text{for all } x \in [a, b].$$

13. Intermediate Value Theorem.

Let f be continuous on a closed interval $[a, b]$. If η is a point between $f(a)$ and $f(b)$ then there is (at least) one point $\xi \in [a, b]$ such that $f(\xi) = \eta$. (Thus, f takes every value between $f(a)$ and $f(b)$ at some point in $[a, b]$.



14. Corollary.

If a continuous function takes values of opposite sign inside a interval, then there exists (at least one) point c in the interval such that $f(c) = 0$.

15. Corollary.

The image of an interval under a continuous function interval is itself an interval. interval.

The Derivative (3 Weeks.)

16. The *derivative* of f at a (if it exists) is

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

17. The process of finding a derivative is called *differentiation*. A function is called *differentiable* at a point a if the derivative $f'(a)$ exists.

18. Claim. f is differentiable at a then f is continuous at a.

    Remark:

    Although a differentible function must be continuous, a derivative need not be. For example: if

    $$f(x) = \begin{cases} x^2\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

    then

    $$f'(x) = \begin{cases} 2x\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0, \end{cases}$$

    is discontinuous at $x = 0$.

19. Claim.

    Let f and g be differentiable functions and let $c, r \in \mathbb{R}$. Then

    (a) $(cf)' = cf'$

    (b) $(f \pm g)' = f' \pm g'$

    (c) $(fg)' = f'g + fg'$

    (d) $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$

    (e) $\dfrac{d}{dx}(c) = 0$

    (f) $\dfrac{d}{dx}x^r = rx^{r-1}$

    (g) $\dfrac{d}{dx}|x| = \begin{cases} 1 & \text{if } x > 1, \\ -1 & \text{if } x < 1. \end{cases}$

20. Trigonometric Functions.

    (a) $\cos' = -\sin$

    (b) $\sin' = \cos$

    (c) $\tan' = \sec^2$

    (d) $\sec' = \sec \cdot \tan$

21. The Chain Rule.

    $$\frac{d}{dx}\left(f(g(x)) = f'\left(g(x)\right)g'(x)\right.$$

    i.e.,

    $$(f \circ g)' = (f' \circ g) \cdot g'$$

22. Related Rates - Example.

23. Implicit Differentiation.

    Curves, defined implicitly by equations of the form

    $$f(x, y) = 0$$

    are usually not the graph of a function. However, in the neighborhood of a point (x, y), the curve may coincide with the graph of a differentiable function. Since this function depends on both x and y its slope y' will, in general, depend on both variables.

    Example:

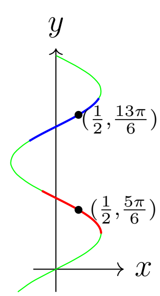
    Consider the curve defined by

    $$x - \sin y = 0$$

    and partially drawn on the right below:

    Implicit differentiation yields:

    $$x = \sin y$$
    $$\Rightarrow 1 = (\cos y)y'$$
    $$\Rightarrow y' = \sec y$$



    In a neighborhood of the point $(\frac{1}{2}, \frac{13\pi}{6})$ the curve coincides with the graph of the differentiable function drawn in blue, whereas, in a neighborhood of $(\frac{1}{2}, \frac{5\pi}{6})$ the curve coincides with (the graph of) a different function drawn in red.

    It happens that in this example the slope y' at a point (x, y) on the curve is a function of y only.

Applications of the Derivative (2 Weeks.)

24. Rolles Theorem.

    If f satisfies all of the following:

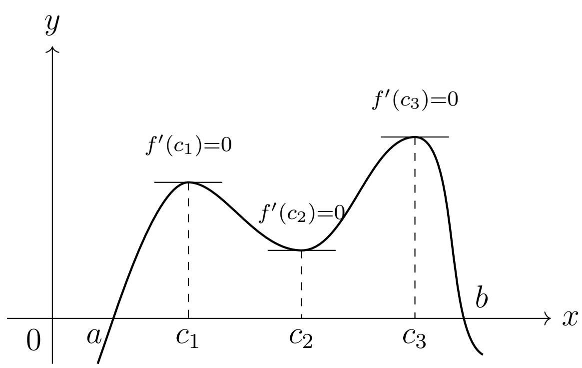
    (a) f is continuous on $[a, b]$

    (b) f is differentiable on (a, b),

    (c) $f(a) = f(b)$

    then, there exists (at least one) $c \in (a, b)$ such that

$$f'(c) = 0.$$



25. Mean Value Theorem.

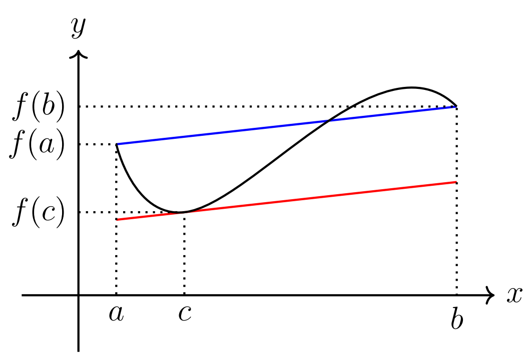
    If f is:

    (a) continuous on $[a, b]$

    (b) differentiable on (a, b)

    then, there exists (at least one) $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



26. Corollary to Mean Value Theorem.

    Let f be continuous on an interval I and differentiable on the interior of I

    (a) If $f'(x) > 0, \quad \forall x \in I$ then f is increasing on I.

    (b) If $f'(x) < 0, \quad \forall x \in I$ then f is decreasing on I.

    (c) If $f'(x) = 0, \quad \forall x \in I$ then f is constant on I..

27. Definition. Let f be defined on an open interval I. A point $a \in I$ is a critical point of f if either $f'(a) = 0$ or $f'(a)$ does not exist.

28. First Derivative Test.

    Relative extrema occur where f' changes sign. More precisely, Let f is continuous at a critical point a. If for some $h > 0$,

$$\begin{cases} f'(x) > 0, & \text{if } a - h < x < a, \\ f'(x) < 0, & \text{if } a < x < a + h. \end{cases}$$

    then f has a relative maximum at a. On the other hand, If for some $h > 0$,

$$\begin{cases} f'(x) < 0, & \text{if } a - h < x < a, \\ f'(x) > 0, & \text{if } a < x < a + h. \end{cases}$$

    then f has a relative minimum at a.

29. Absolute Maxima and Minima.

    Let f be continuous on a closed interval. Then the absolute maxima or minima of f are either critical points or endpoints of the interval.

30. Definition.

    A differentiable function f defined on an interval I is

    (a) *convex* if f' is (strictly) increasing on I.

    (b) *concave* if $-f$ is convex.

31. Let f be twice differentiable on an open interval I. Then

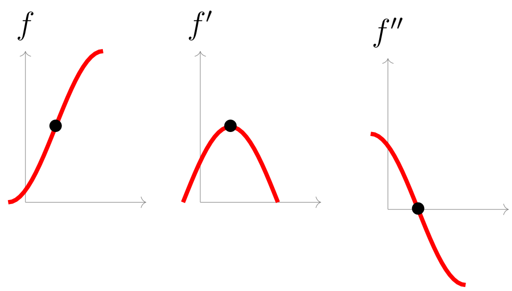
    (a) If $f'' > 0$ on I then f is convex.

    (b) If $f'' < 0$ on I then f' is concave.

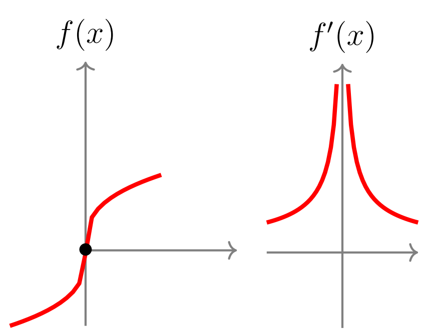
32. Let f be continuous on an interval I. A point $a \in I$ is an *inflexion point* if,
    (a) f has a tangent at a and
    (b) f changes concavity at a.

33. Let f be differentiable on an open interval I. If $a \in I$ is a point of inflexion and $f''(a)$ exists then necessarily $f''(a) = 0$.

34. Corollary. Let f be differentiable on an open interval I. If $a \in I$ is an inflexion point, then either $f''(a) = 0$ or $f''(a)$ does not exist.



35. A function f is said to have *vertical tangent* $x = a$ if either
$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = +\infty$$
or
$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = -\infty.$$



36. Second Derivative Test.
    Let f be differentiable on an open interval I and let $a \in I$. If
    (a) $f'(a) = 0$ AND
    (b) $f''(a) > 0$

    then f has a local minimum at a.

37. Optimization with Constraints

Antiderivatives (2 Weeks.)

38. Definition. A function F such at $F' = f$ is called either an *antiderivative* or *primtive* or *indefinite integral* of f.

39. If F and G are two antiderivatives of f on an interval I then, by the mean value theorem, $F = G + c$ where c is constant.

40. The set of antiderivatives of f is denoted by the (imprecise but convenient) notation
$$\int f(x)\, dx + c$$
    or simply
$$\int f + c.$$
    Hence,
$$\left(\int f + c\right)' = f$$

41. $\displaystyle\int f' = f + c$

42. $\displaystyle\int x^p dx = \frac{x^{p+1}}{p+1} + c; \quad p \neq -1$

43. If $F' = f$ then $\displaystyle\int f = F + c$

44. Linearity.
    (a) $\displaystyle\int (f+g) = \int f + \int g$
    (b) $\displaystyle\int cf = c \int f$

45. Integration by Parts.
$$\int fg' = fg - \int f'g \tag{1}$$

46. Direct Substitution.

$$\int (f \circ g) \cdot g' = F \circ g \tag{2}$$

where $F' = f$.

47. Inverse Substitution.

$$\int f = \left(\int (f \circ g) \cdot g' \right) \circ g^{-1} \tag{3}$$

48. Laissant's Formula.

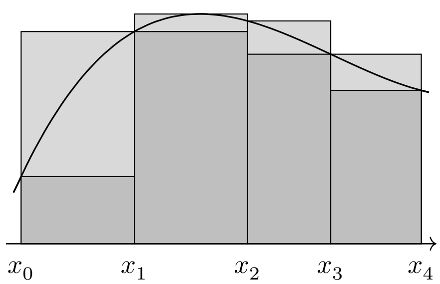
$$\int f^{-1}(y)\, dy = y f^{-1}(y) - F \circ f^{-1}(y) + c$$

49. Formulae for antiderivatives.

The Definite Integral (2 Weeks.)

50. Sigma Notation.

51. Upper and Lower Sums.



52. The Riemann Integral.

53. The Fundamental Theorem of Calculus (FTC).

Let f be continuous on the closed interval $[a, b]$. Define F by

$$F(x) = \int_a^x f(t)\, dt.$$

Then F is continuous on $[a, b]$ and is an antiderivative of f on (a, b), i.e.,

$$F'(x) = f(x), \quad \forall x \in (a, b)$$

54. Corollary.

Definite integrals can be evaluated by antiderivation. Specifically:

If f is continuous on $[a, b]$ and G is an antiderivative of f in $[a, b]$ then

$$\int_a^b f(t)\, dt = G(b) - G(a)$$

55. Integration by Parts.

$$\int_a^b f(x) g'(x)\, dx = [f(x)g(x)]_a^b - \int_a^b f'(x) g(x)\, dx$$

56. Change of Variables Theorem.

$$\int_{\varphi(a)}^{\varphi(b)} f(x)\, dx = \int_a^b f(\varphi(t))\, \varphi'(t)\, dt.$$

57. Parity Considerations.

58. Applications of the Definte Integral.

Applications. (3 Weeks.)

59. Trivial linear ODE's:

If g is continuous, the initial value problem

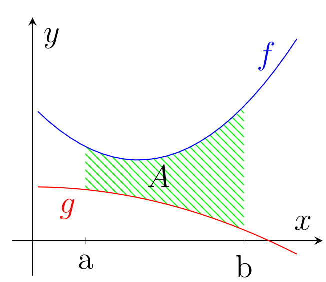
$$x' = g(t); \quad x(t_0) = x_0$$

has unique solution:

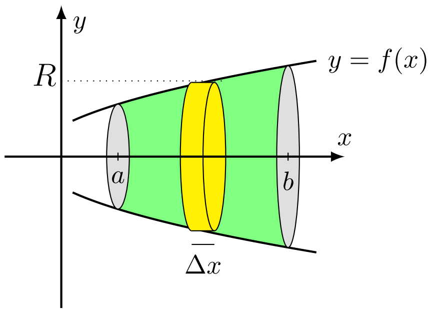
$$\phi(t) = x_0 + \int_{t_0}^t g(\tau) d\tau$$

60. The area between two curves.

$$A = \int_a^b |f(x) - g(x)|\, dx$$



61. Computation of Volumes using transverse sections and other methods.



62. Methods of Defining Functions

63. Definition of the natural logarithm function.

    (a) $\ln x = \displaystyle\int_1^x \frac{1}{t}\,dt$

    (b) The exponential function.

$$\exp = \ln^{-1}$$

64. Exponential growth.

65. First order separable differential equations.

    If f and g are continuous and $g(y_0) \neq 0$ and $f(x_0) \neq 0$ then in a neighborhood of (x_0, y_0) there exists a unique solution ϕ of the initial value problem

$$y' = \frac{g(y)}{f(x)}; \quad y(x_0) = y_0$$

    given implicitly by

$$\int_{y_0}^{\phi(x)} \frac{1}{g(\eta)} d\eta = \int_{x_0}^{x} \frac{1}{f(\xi)} d\xi.$$

Instructional Strategies

Readings, online videos, lectures, class discussion, exercises.

El Departamento de Matemáticas ofrece un servicio gratuito de tutorías para todos los cursos básicos del área. Los tutores son Ayudantes de Cátedra, estudiantes de los programas de Maestría y Doctorado del Departamento de Matemátics. El servicio está disponible de lunes a viernes en el salón C-208, y no requiere cita previa.

Contingency Plan in Case of Emergency

In the event of a closure for any reason, the course will be continued via **Google Meet** and Moodle. (Go here – if Moodle is down.)

Grading Scale

Letter grade (A, B, C, E or F)

Evaluation

The grade will be based upon three exams (300 points) and one comprehensive final exam (200 points). The worst exam of 100 points will be eliminated. Makeup exams and extra credit will not be given. The grading scale will be no worse than the following: A: $90 - 100$; B: $80 - 89$; C: $65 - 79$; D: 60-64; F: < 60. Copying or other forms of cheating will result in an automatic F for the course. Calculators and electronic apparati may not be used. In accord with UPR regulations, persistent lateness or unexcused absence from class may result in a lowered or failing grade and loss of financial support.

Texts

1. George B. Thomas Jr. Maurice D. Weir, and Joel R. Hass, *Calculus*, 13th Edition, Pearson 2014. –The official text.

2. Serge Lang, A First Course in Calculus, Addison Wesely, 1968. –Clear introduction to Calculus written by a great mathematician

3. Salas, Hille and Anderson, Calculus, One and Several Variables, John Wiley.

Bibliography

1. Mary P. Dolciani, Robert H. Sorgenfrey, John A. Graham, David L. Myers, *Introductory Analysis*, Houghton Mifflin, 1987. –a very well written introduction to precalculus and calculus at a higher mathematical level than the usual precalculus texts.

2. Kunihiko Kodaira, *Basic Analysis, Japanese Grade 11*, American Mathematical Society, 1991. –A very good highschool calculus text from Japan.

3. Serge Lang, A First Course in Calculus, Addison Wesely, 1968. –Clear introduction to Calculus written by a great mathematician

4. Serge Lang, A Second Course in Calculus, Addison Wesely, 1968.

5. David A. Santos, *Precalculus*, Open Math Text, 2010.

   The great price (and weight) of textbooks bears no relation to their academic quality. The very excellent *Elementary Algebra* and *Precalculus* texts by David Santos can be legally downloaded for free. They contain all the material needed for this course.

6. David A. Santos, *Pre- y Cálculo Criollos*, Open Math Text, 2008.

7. Ho Soo Thong, Tay Yong Chiang and Kah Khee Meng, *College Mathematics Syllabus C, Volumes 1 and 2*, Second edition (1989), Pan Pacific Publications, ISBN-9971-63-931-9.

   – These are excellent comprehensive and carefully written texts. They are well worth buying since they contains much good material relevant to a number of other courses including physics and statistics.

8. Michael Spivak, *Calculus*, Cambridge University Press, 2006.

   –Spivak's textbook is one of the finest introductions to mathematical analysis. The book is ideal for honours students and mathematics majors.

9. Ian Stewart and David Tall, *Foundations of Mathematics*, Oxford University Press, 1977.

   –As its title suggests, the book *Foundations of Mathematic* provides a rigorous introduction to the foundations of Mathematics. It shows how the real numbers can be constructed from the axioms of Peano. Any student who is serious about achieving a good level in Mathematics should study the material in this book. It is essential that a math or computer science major know the material in this book.

Academic Honesty

All homework should be done independently –collaboration is not permitted. Cheating and other anti-intellectual behavior may result in an F. Please make sure you read, understand and abide by the Academic Integrity Code of the University of Puerto Rico.

Students with Disabilities

If you have a disability for which you may be requesting an accommodation, please contact both your instructor and the office of Vocational Rehabilitation as early as possible in the term. Vocational Rehabilitation will verify your disability and determine reasonable accommodations for this course.

Regulations on discrimination (Certification 39, 2018-2019)

"The University of Puerto Rico prohibits discrimination based on sex, sexual orientation, or gender identity in any of its forms, including that of sexual harassment. According to the Institutional Policy Against Sexual Harassment at the University of Puerto Rico, Certification Num. 130, 2014-2015 from the Board of Governors, any student subjected to acts constituting sexual harassment, must come to the Office of the Student Ombudsperson, the Office of the Dean of Students, and/or the Coordinator of the Office of Compliance with Title IX for an orientation and/or a formal complaint".