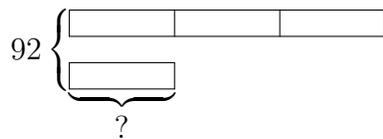


1. Prealgebra (arithmetic with letters).
2. We represent numbers by letters to:
  - (a) Express unknowns
  - (b) Express identities
  - (c) Express relationships
  - (d) Calculate in the abstract.
3. Example. There are three times as many elephants as tigers. The total number of elephants and tigers is 92. How many tigers?

(a) Solution by diagram



$$4 \text{ units} = 92$$

$$1 \text{ unit} = 92 \div 4 = 26$$

(b) Solution by algebra

Let  $x$  be the number of tigers.  
 Then  $3x$  is number of elephants.  
 $x + 3x = 92$   
 $4x = 92$   
 $x = 92 \div 4 = 26$

4. For large numbers diagrams are not convenient.  
 Example. Juan has 16 times more money than Paco. Together they have \$128. How much money does Juan have?
5. A *numerical expression* is a combination of numbers, parentheses and operation symbols representing a single number.
6. An *algebraic expression* is a combination of numbers, parentheses operation symbols and letters which represents a single number when suitable numbers

are substituted for the letters.

Example:  $\frac{x + 3y}{x^2 - 8}$

7. Every numerical expression is an algebraic expression.
8. Set models of algebraic expressions such as:
  - (a)  $6a - 2a$
  - (b)  $3a + 5 + 2a - 3$
9. Measurement and area models of algebraic expressions such as  $(x^2 + 3x + 4) + (2x^2 + x + 7)$
10. An *equation* is a declaration that two expressions are equal.
  - (a)  $x + 3x = 96$
  - (b)  $x = y + 2$
11. An equation is called an *identity* if it is true no matter which numbers the letters in the equation represent.
12. Area interpretation and formal proofs of the identities:
  - (a)  $(a + b)(a + b) = a^2 + 2ab + b^2$
  - (b)  $(x + 1)^2 = x^2 + 2x + 1$
  - (c)  $(a + b)(c + d) = ac + bc + ad + bd$
13. Proof and interpretation of the identity
 
$$(a - b)(a + b) = a^2 - b^2$$
14. Applications such as:
  - (a)  $14^2 = (10 + 4)^2 = 10^2 + 80 + 16 = 196.$
  - (b)  $19 \times 21 = (20 - 1)((20 + 1) = 20^2 - 1^2 = 399.$
  - (c)  $14 \times 18 = (16 + 2)(16 - 2) = 16^2 - 2^2.$