

EXERCISES ON THE NORMAL DISTRIBUTION

Philip Pennance

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1. Let F be the distribution function of a continuous random variable. If $F(3.24) = .8765$ and $F(3.25) = .8805$, estimate the probability that $X < 3.242$. Show your reasoning and state any assumption made.
2. A random variable X has distribution $N(0, 10^2)$. What percentage of the data falls in each of the following intervals:
 - (a) $-2\sigma < X < 2\sigma$.
 - (b) $-3\sigma < X < 3\sigma$.Compare the values obtained with those predicted by Chebychev's theorem.
3. Find the percentiles P_{25} , P_{50} , P_{75} , P_{90} for a random variable X
 - (a) with standard normal distribution.
 - (b) with mean 100 and variance 16.
4. Let X be a random variable with distribution $N(10, 2^2)$. Let F be the distribution function of X . Find
 - (a) $F(11)$
 - (b) $F(6)$
 - (c) $F(P_{20})$ where P_{20} is the 20th percentile.
 - (d) $F^{-1}(.2)$
5. A manufacturer sends a shipment of 2000 items to a client. Suppose that the client will not accept the items if at least 7% of a sample of size 100 are defective. Find:
 - (a) The probability that the client accepts the shipment if 10% of the items are defective.
 - (b) The probability that the client does not accept the shipment if 5% of the items are defective.
6. Suppose that 80% of the PNP support Rosselló. In a sample of size 100 calculate the probability that at least 70% support Rosselló.
7. A lathe is designed to produce pieces with dimension $X \sim N(2, .005^2)$. The lathe stops when $x > 2.01$ or $x < 1.99$.
 - (a) The probability that the machine stops after producing one piece if it is functioning correctly (that is, if the pieces that are produced are normally distributed with mean 2 and variance 0.005^2).
 - (b) The probability that the machine stops after producing one piece if $X \sim N(1.99, .005^2)$.
 - (c) The probability that the machine continues after producing one piece if $X \sim N(1.99, .005^2)$.
8. The random variable arm length of pilots has distribution $X \sim N(48, 2^2)$. Find the distance between the pilot and the controls that an airplane must have so that at least 95% of the pilots have the controls within their reach.
9. In a Math 3026 exam, the mean of a group of students is 80 with standard deviation 3. A professor, with nothing better to do, calculates that a certain student has a standardised score Z of 2.5. Find the student's score on the exam. If the scores were normally distributed, approximately what percentage of standardised scores would have $-2 < Z < 2$.

10. Let X be a random variable with distribution $N(10, 2^2)$. Let F be the distribution function of X . Find
 - (a) $F(11)$
 - (b) $F(6)$
 - (c) $F(P_{20})$ where P_{20} is the 20th percentile.
 - (d) $F^{-1}(.2)$
11. Let X and Y be random variables with $\sigma(X) = 2$, $\sigma(Y) = 3$, $\mu(X) = 100$, $\mu(Y) = 200$. Find the variance of $2X + Y$:
 - (a) if X and Y are independent
 - (b) if the coefficient of linear correlation $\rho(X, Y) = 0$
 - (c) if the covariance $\text{cov}(X, Y) = 2$
12. The standard normal distribution has density function $f(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$
 - (a) What is the meaning of the area under the graph of f between any 2 values of x ?
 - (b) Find the maximum point of f .
 - (c) Find the inflection points of f .
 - (d) Show that the graph of f is symmetric with respect to the vertical axis.