

EXERCISE ON HYPOTHESIS TESTING

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1. A coin is tossed 10 times and $X = 10$ heads obtained. Calculate the p -value for this outcome. Test, at the significance level $\alpha = 0.01$, the hypothesis that the coin is biased. What is the probability of a type I error?
2. A coin is tossed 64 times and $X = 36$ heads obtained. Test, at the significance level $\alpha = 0.05$, the hypothesis that the coin is biased.
3. Suppose that 54% of the population of Puerto Rico support the governor before his announcement of a plebiscite. After announcing the plebiscite a random sample of 1000 people is found to contain 480 who support the governor. Is there evidence, at the 0.05 level, of a change of opinion?
4. Let X be a random variable having a triangular shaped density function with st. dev. σ . Find the p -value for the test:

$$\begin{aligned} H_0 : \mu(X) &= m \\ H_A : \mu(X) &< m \end{aligned}$$

given that in a random sample of size $n > 30$ values of X ,

$$\frac{\bar{X} - m}{\frac{\sigma}{\sqrt{n}}} = 1.48$$

5. Let X be a random variable having a density function which is semicircular in form and st. dev. = 3.5. Consider the hypothesis test:

$$\begin{aligned} H_0 : \mu(X) &= 20.0 \\ H_A : \mu(X) &> 20.0 \end{aligned}$$

with significance level $\alpha = 0.05$. If a sample of size 50 is used, find the minimum value of the sample mean at which the null hypothesis would be rejected.

6. In 1981 the mean G.P.A. of Math 3026 students was 2.10. In 2011, a sample of 36 students had a G.P.A of 2.30. with st. dev. of 0.60. Is there evidence, at the 0.01 level, of an increase in G.P.A?
7. The data values 33, 42, 38, 37, 30, 42 were taken from a population with normal distribution. Is there evidence, at the 0.05 level that the mean is greater than 35?
8. Explain why the probability of a type I error in a statistical hypothesis test is equal to the significance level.
9. In the following table, n represents sample size, s^2 sample variance, and $\bar{X}$, the mean exam score in a Departmental exam given to two sections of Math 3026. Section I is taught by a regular professor and II by an adjunct. Both sections take the same exam.

	n	$\bar{X}$	s^2
Section I	49	2.2	.49
Section II	81	2.0	.36

Is there evidence, at the 0.01 level, of a difference in the grades given by the professor and the adjunct? What factors might cause such a difference?

10. Independent samples are taken from each of two normal populations with $\sigma = 5$.

Sample	n	$\bar{X}$
I	32	104
II	40	110

$\sigma = 5$. Is there evidence, at the 0.01

level, that the means of the two populations are equal?

11. The data below shows boiling times for two types of water heater.

Heater A: 4.7, 5.1, 5.5, 5.7, 4.0

Heater B: 4.2, 4.2, 4.7, 4.5, 4.4

Is there evidence, at the 0.05 level, of a difference in efficiency. Assume both populations are normal.

12. A professor of Math 3026, in search of a miracle, attempts to construct an exam in which the grades are normally distributed with standard deviation at

least 20 points. He gives the exam to a sample of 21 students and finds that the scores are normally distributed with standard deviation $s^2 = 15$. Is there evidence, at the 0.05 level, that the population variance is insufficient?

13. An aspirin manufacturer requires his tablets contain $X = 500\text{mg}$ of aspirin with standard deviation $\sigma = 10\text{mg}$. A sample of 51 tablets is found to have $s = 12\text{mg}$. Is there any evidence, at the 0.05 level, that the standard deviation is greater than 10mg? Assume that X has a normal distribution.