

Math 3026–Additional Exercises
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1. Suppose that A and B are events with $P[A] = 3/5$ and $P[B] = 3/10$. Find:
 - (a) $P[A \cup B]$ if $P[A \cap B] = 2/5$.
 - (b) $P[A \cup B]$ if A, B are mutually exclusive.
 - (c) $P[A|B]$ if $P[A \cap B] = 1/5$.
2. Three 6-sided dice, A, B , and C have faces labelled as follows: A : 1 1 4 4 4 4, B : 3 3 3 3 3 3, C : 2 2 2 2 5 5.
 - (a) If A and B are thrown, show that on average A wins. i.e., show that $P[A \text{ wins}] > P[B \text{ wins}]$.
 - (b) If B and C are thrown show that $P[B \text{ wins}] > P[C \text{ wins}]$.
 - (c) Can you conclude from the above that $P[A \text{ wins}] > P[C \text{ wins}]$.
 - (d) Show that, if B and C are thrown then, in fact, $P[C \text{ wins}] > P[A \text{ wins}]$.
3. Two 6 sided dice are thrown. Let X be the number shown on the face of the first die and Y the second. Indicate the sample space. Find the following probabilities.
 - (a) $P[X + Y = 7]$.
 - (b) $P[X > 4]$.
 - (c) $P[X + Y = 7|X > 4]$.Are the events $X + Y = 7$ and $X > 4$ independent. Justify your answer.
4. A box contains three balls (1 yellow, 1 green and 1 blue). Consider the experiment of repeatedly removing and replacing a ball until a blue one is obtained.
 - (a) Construct a tree diagram showing the first four repetitions
 - (b) Find the probability that the experiment will terminate in exactly 2 repetitions.
5. A box contains 5 balls, two with the number 1, two with the number 2 and one with the number 3. Two balls are selected with replacement.
 - (a) Let \bar{X} be the mean of the two balls selected. Find the density function of the random variable \bar{X} .
 - (b) Find the mean of \bar{X} .
 - (c) Find the mean of variance \bar{X} .
6. A box contains 3 red balls, 4 blue balls and 1 green ball. Two balls are selected without replacement. Find the probability that:
 - (a) Both are red.
 - (b) Both are the same color.
 - (c) That one is green and one is red.
 - (d) That the first is green and the second is red.
7.
 - (a) Find an example of two events which are independent but not mutually exclusive.
 - (b) Find an example of two events which are mutually exclusive but not independent.
8. In a certain game a player selects one of three boxes without opening it. One of the boxes contains a prize of \$1000 and the other two are empty. A judge, who knows the location of the prize, opens one of the two boxes not selected by the

player and offers him the opportunity to change his selection.

Find the probability that the player will

wins the prize if he

(a) changes his selection.

(b) does not change his selection.