

MATH 3026 PRACTISE FOR EXAM 1

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1. For the data in the following stem and leaf plot find:

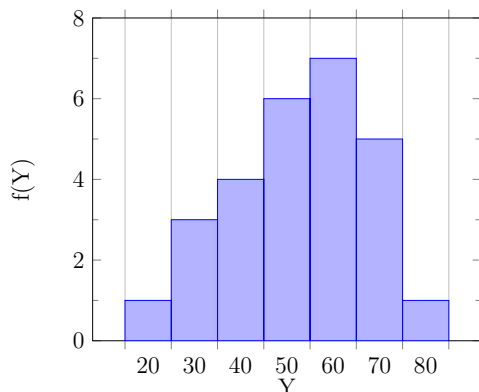
- (a) P_{25} .
- (b) The mean
- (c) Q_3 .
- (d) $F(Q_3)$ where F is the empirical distribution function
- (e) Explain why $F(Q_3)$ in (d) is approximately 0.75.

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|3|
|4|2297
|5|33422
|6|73
|7|3397
|8|253
    
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2. For the variable Y in the following frequency histogram

- (a) Find $P[Y = 60]$.
- (b) Find the mean Y .
- (c) Find the median P_{50}
- (d) Find the mode
- (e) Find P_{25} .
- (f) Sketch the distribution function.



3. The following data show male and female shark lengths. Draw a back to back stem and leaf plot. What do you observe.

53 m, 53 m, 42 m, 67 f, 73 m, 54 m, 82 f, 71 f, 42 f, 49 m, 79 m, 52 m, 63 m, 85 f, 47 m, 73 f, 83 f, 52 f.

4. Suppose that repeated measurements of a variable X yield the frequencies given in the table:

x	0	1	2	3	4	5
f(x)	1	3	5	5	2	1

- (a) Find the empirical probability that $2 \leq X \leq 4$
- (b) The mean $\bar{X}$.
- (c) Sketch the empirical distribution function for the variable X . Label the axes correctly.

5. For the sequence $(3, 4, 2, 10, -5)$. Find:

- (a) $\sum_{i=1}^5 X_i$
- (b) $\sum_{p=3}^5 X_p$
- (c) $\sum_{i=1}^8 7$
- (d) $\sum_{i=1}^5 X_i^2$
- (e) $\left[\sum_{i=1}^5 X_i \right]^2$

6. Let $S_{XX} = \sum_{k=1}^n (X_k - \bar{X})^2$.

- (a) Calculate S_{XX} for the following data values of X : 2, 3, 5, 7, 1, 3, 4.

(b) Show that

$$S_{XX} = \sum_{k=1}^n X_k^2 - \frac{\left(\sum_{k=1}^n X_k\right)^2}{n}$$

7. A sample of 72 values of a random variable yields $\bar{X} = 40$ $s^2 = 100$. Find:

(a) $\sum_{i=1}^{72} X$ (b) $\sum_{i=1}^{72} X^2$

8. The SAS instruction

$$X = 5 * RANUNI(98765) + 3;$$

produces random numbers with probability density function

$$f(x) = \begin{cases} 1/5 & \text{if } 3 < x < 8, \\ 0 & \text{otherwise} \end{cases}$$

Find the following probabilities:

- (a) $P(X < 5)$.
 - (b) $P(4 < X < 6)$.
 - (c) $P(X < 2)$.
 - (d) $P(X = 5)$.
 - (e) $P(X < 12)$.
9. Group the following data by means of a suitable step function and draw a histogram of the grouped values 24, 32, 35, 37, 42, 43, 46, 49, 50, 51, 53, 53, 56, 58, 61, 61, 63, 64, 69, 69, 73, 74, 76, 76, 79, 83.

10. Calculate the standardized value Z for each member of the population $\{2, 4, 6\}$. Verify by direct that $EZ = 0$ and $\text{Var}Z = 1$.
11. A random variable X has mean 90 and standard deviation 10. Find the linear function which converts the grades to a scale with mean 70 and standard deviation 2.
12. The following table shows the results of a group of students in two exams.

	Examen I	Examen II
Mean	83	78
Variance	4	1

A student obtained a score of 90 points in exam I and 80 in exam II. In which exam did he have the highest percentile? State any assumptions you make.

13. Show that length is a random variable of ratio type. Hint. The solution is contained in [this article](#).
14. Show that the time of a telephone call is an interval random variable.
15. Find the effect on EX , σ_X , $\text{Var}(X)$ if:
- (a) 10 is added to every observation.
 - (b) Every observation is multiplied by 2
 - (c) Every observation is first multiplied by 2 and then 10 added
 - (d) 10 is added to every observation and the result multiplied by 2