Math 3026–Additional Exercises
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1. A random element is selected from $S = \{1, 3, 5, 7\}$. Let $X : S \to R$, be defined by $X(\omega) = \omega$. Find the density function of the random variable X.

2. Consider the function f determined by the table:

x	$f(x)$
2	1/4
4	1/8
6	1/2
1	1/8

   (a) Show that f is a probability density function.

   (b) Find the corresponding mean and variance.

3. A random variable X has probability density function

   $$f(t) = (2t - 3)/9, \quad t \in \{2, 3, 4\}$$

   (a) Find the mean of X

   (b) Find the variance of X

   (c) Find the mean of X^2

4. Consider the sample space for the random experiment of tossing two coins, namely, $\Omega = \{HH, HT, TH, TT\}$. Let $X : \Omega \to \mathbb{R}$ be the number of heads obtained, f the density function of X, and P the probability function.

   (a) Calculate the mean of X using the formula

   $$EX = \Sigma\{eP(e) : e \in \Omega\}$$

   (b) Calculate the mean of X using the the density function of X.

   (c) Make sure that the results of (a) and (b) are the same.

5. Let $g(x) = \sqrt{x - 1}, \quad x \geq 1$.

   (a) Find a formula for the inverse function $g^{-1}(x)$.

   (b) A random variable X has density function

   $$f(x) = (2x - 3)/9, \quad x \in \{2, 3, 4\}$$

      i. Find an equation for the density function of the random variable $Y = \sqrt{X - 1}$.

      ii. Find the mean and standard deviation of Y.

6. A random variable X has density function: $f_X : \{-4, -3, -2, -1, 1, 2, 3, 4\} \to [0, 1]$ given by $f_X(t) = |t|/20$

   (a) Find the density function of $Y = X^2$.

   (b) Draw a histogram of the distribution of Y.

   (c) Find the mean of Y.

   (d) Find the probability that $Y < 3$.