Math 3026–Additional Exercises

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In each of the following carefully prove your answer by reference to the appropiate combinatorial principle.

1. A multiple choice exam has 20 questions each with 5 possible answers. In how many ways can this exam be answered.?

2. (a) In how many ways can a committee of 2 persons be chosen from a set of 7 persons?
   (b) In how many ways can a president and a vicepresident be selected from a committee of 7?

3. Suppose there are 6 roads between San Juan and Caguas, 3 roads between Caguas and Ponce, and 4 roads between Ponce and Jurutungo. How many ways are there to travel between San Juan an Jurutungo?

4. In how many ways can a team of ten persons be divided into two teams of five.

5. In how many ways can 5 men and 5 women be seated in a row of 10 chairs if no two persons of the same sex are in adjacent chairs.

6. For each natural number k let $\mathbb{N}_k$ denote the set $\{n \in \mathbb{N} : 1 \leq n \leq k\}$.
   (a) Find the number of functions from $\mathbb{N}_5$ to $\mathbb{N}_{365}$.
   (b) Find the number of injective functions from $\mathbb{N}_5$ to $\mathbb{N}_{365}$.
   (c) Find the probability that in a set of 5 people there is a pair with the same birthday.

7. How many words of three letters can be formed from the alphabet $\mathcal{A} = \{a, b, c, d, e, f\}$?

8. How many words of three letters containing exctly one e can be formed from the alphabet $\mathcal{A} = \{a, b, c, d, e, f\}$?

9. Let $B_2 = \{00, 01, 10, 11\}$ the set of binary numbers of length 2. In propositional logic a function from B_2 to $\{0, 1\}$ is called a *truth table* For example, the concept of *implication* is expressed by the truth table $f : B_2 \to \{0, 1\}$ given by $f(1, 1) = 1$, $f(1, 0) = 0$, $f(0, 1) = 1$, $f(0, 0) = 1$. Find the number of different truth tables.

10. A DNA chain is a word in the alphabet $\{A, G, C, T\}$. Find the number of words of length 14 with
    (a) 4 A's and 10 G's.
    (b) 4 A's and 5 G's, 2C's and 3T's
    (c) C at one end and A or G at the other.
    (d) C at one end and A or G at the other and with exactly 3 A's.?
    (e) At least 12 consecutive A's.

11. A library contains 6 books in French, 5 in Spanish and 8 in English.

    (a) How many ways can a subset of two books be chosen if they have different languages.

    (b) How many ways can a subset of three books be chosen if exactly two are in French.