

EXERCISES ON THE DISTRIBUTION OF THE MEAN

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February 2010

1. A random variable X represents the height of adults of a certain species of ape. Suppose $X \sim N(5, 2^2)$. A random sample of 16 apes is taken. Find the following probabilities:
 - (a) $P[\bar{X} > 8]$
 - (b) $P[\bar{X} > 10]$
 - (c) $P[6 < \bar{X} < 7]$
2. A random element is selected from the set $S = \{1, 3, 5, 7\}$. Let X be the number selected. Find the density function and distribution function of the variable X .
3. A random sample of size two is taken with replacement from $S = \{1, 3, 5, 7\}$. Let $\Omega = S \times S$. Define random variables $X_1, X_2 : S \times S \rightarrow R$ by $X_1(i, j) = i$ and $X_2(i, j) = j$. i.e. X_1 and X_2 yield the results of the first and second selections respectively.
 - (a) Let $\bar{X} = (X_1 + X_2)/2$ the sample mean. Find the density functions for X_1 , X_2 and $\bar{X}$.
 - (b) Are the random variables X_1 and X_2 independent? Explain.
 - (c) Are the random variables X_1 and X_2 identically distributed? Explain.
 - (d) Find the population mean.
 - (e) Is $\bar{X}$ an unbiased estimator of the population mean? Justify your answer.
4. A population has mean 60. Find the variance if 10% of samples of size 100 have mean $\bar{X} > 65$
5. Suponga que la variable aleatoria X tiene la distribución $N(\mu, \sigma^2)$. Halle la distribución de:
 - (a) $X - \mu$
 - (b) $\bar{X}$
 - (c) $\bar{X} - \mu$
 - (d) $\sqrt{n}(\bar{X} - \mu)/\sigma$