Truth in Propositional Calculus

Philip Pennance[1] – February 1998.

1. Introduction.

   In the calulus of propositions (i.e. logic without quantifiers), if p, q are propositions, then so are the *compound* statements: $p \vee q$ (*disjunction*); $p \wedge q$ (*conjunction*); $p \to q$ (*implication*); $p \leftrightarrow q$ (*equivalence*).

   The *truth* of these statements is determined by the truth of p and q according to the following tables known as *truth tables* in which 1 denotes *true* and 0 denotes *false*. A complete list of truth tables for the case two propositions can be found here.

2. Definition.

   The disjunction $p \vee q$ is is false if both p and q are false, and true otherwise.

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

3. Example.

   If p is the statement "I teach mathematics" and q is the statement "I teach physics" then $p \vee q$ is the statement "Either I teach mathematics or I teach physics".

4. Remark. Since disjunction of p and q is true when both p and q are true $\vee$ is sometimes called the *inclusive or*.

5. Definition.

   The conjunction $p \wedge q$ is true if both p and q are true and false otherwise.

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

   The conjunction $p \wedge q$ is true if both p and q are true and false otherwise.

6. Example

   If p is the the statement "4 is positive" and q is the statement "4 is less than 5", then $p \wedge q$ is "4 is positive and less than 5".

7. Definition.

   The statement "p implies q" is *false* if p is true and q is false and true otherwise.

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

8. Warning.

   The truth of a statement such as

   "*the presence of oxygen implies fire*"

   can **NOT** be determined solely from a truth table. Empirical knowlede is required.

9. Definition.

   The *equivalence* $p \leftrightarrow q$ is true if p and q have the same truth values and false otherwise.

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

   Linguistically $\leftrightarrow$ corresponds to the phrase "if and only if".

[1] https://pennance.us