

Statements and Truth in Mathematics

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1. Introduction.

Long ago, Pilate asked the important question “*What is truth*” (John 18:38). Here we are concerned with mathematical rather than religious truth. However, in mathematics, the answer is not simple.

2. Definition.

A sentence which possesses a truth value (true or false) will be called a *statement*.

As the following examples reveal mathematical statements may be true for a variety of reasons.

3. Examples:

(a) *The triangle has three sides.*

This is true by [definition](#) of a triangle.

(b) $\sqrt{2}^{\sqrt{2}}$ *is either rational or irrational.*

This is a logical truth (tautology). It is true because of the meaning of the [logical connective](#) *or*.

(c) *Given a line L , and a point P not on L , there is exactly one line through the point P parallel to the line L .*

Mathematical theories contain statements which we do not attempt to prove but merely assign truth to them. Such statements are called *axioms*. The above statement is an axiom of, and hence true in Euclidean geometry. Its truth is contingent on the geometry being Euclidean.

(d) *The sum of the angles in any tri-*

angle is 180 degrees.

Statements deduced from axioms by a mathematical proof using [valid arguments](#) are called *theorems*. this is a theorem of, and hence true in Euclidean geometry. It may well be false in other geometries.

(e) *Every even integer greater than 2 can be written as the sum of two primes.*

This is a conjecture of Goldbach (1742). Until it is either proven or unproven no truth value can be sensibly assigned.

4. Remark.

Mathematical truth is not the same as empirical truth. Mathematical statements cannot be proven by experiment in the real world (say by measuring the angles of a triangular object).

5. Warning.

Not everything that we say is a statement in the sense that a truth value can be assigned. Consider the set T of all true statements uttered in the Senate of Puerto Rico. A senator says: “*The statement that I am now uttering is not in T* ”. His utterance cannot be considered a statement since if it were true (in T) then it contradicts itself and, similarly, if it were false it also contradicts itself. Thus, no truth value can be assigned.

6. Concluding Remark.

For a rigorous discussion of the concept of truth in logic, the ambitious reader should (one day in the future) read [Tarski's truth definitions](#).

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