University of Puerto Rico - Río Piedras
Department of Mathematics

MATE 3018 – EXAM III – Semester I - 2015

Instructions: *A valid photo ID should be available if requested. The use of electronic and other aids or devices is strictly forbidden. Answer all questions. All work must be shown to obtain credit. Continue answers on the back of the page if necessary.*

Last Names: ____________ ____________ First Name: ____________

Student # : __________ Professor: Philip Pennance Section: __________

1. Si $\log x = 2$, $\log y = 1$ y $\log z = 3$ halle:

(a) $\log(xy^5)$

Solution.

$$\begin{aligned} \log(xy^5) &= \log x + \log y^5 \\ &= \log x + 5\log y \\ &= 7. \end{aligned}$$

(b) $\log\left(\sqrt{\dfrac{x}{yz}}\right)$

Solution.

$$\begin{aligned} \log\left(\sqrt{\frac{x}{yz}}\right) &= \tfrac{1}{2}\log\left(\frac{x}{yz}\right) \\ &= \tfrac{1}{2}\left[\log x - \log(yz)\right] \\ &= \tfrac{1}{2}\left[\log x - (\log y + \log z)\right] \\ &= \tfrac{1}{2}\left[2 - (1+3)\right] \\ &= -1. \end{aligned}$$

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2. (a) Halle el valor de k tal que las funciones $g(x) = 2^x$ y $f(x) = e^{kx}$ son son iguales para todo $x \in \mathbb{R}$.

Solution.

$$\begin{aligned} & 2^x = e^{kx} \\ \iff\ & \ln 2^x = kx \text{ for all } x \\ \iff\ & x\ln 2 = kx \text{ for all } x. \end{aligned}$$

Putting $x = 1$ yields $k = \ln 2$.

(b) Halle números reales a y b tal que

$$\log\left(\frac{x^2+2x+1}{25}\right) = a\log(x+1) + b$$

Solution.

$$\begin{aligned} & \log\left(\frac{x^2+2x+1}{25}\right) = a\log(x+1) + b \\ \iff\ & \log(x+1)^2 - \log 25 = a\log(x+1) + b \\ \iff\ & 2\log(x+1)^2 - \log 25 = a\log(x+1) + b \end{aligned}$$

Putting $x = 0$ shows that $b = \log 25$.
Putting $x = 2$ then yields

3. Resolver para x:

(a) $\log x = 5$

Solution.

$x = 10^5$

(b) $\left[4^{2x-1}\right]^{-2} = 16^x$

Solution.

$$\begin{aligned}\left[4^{2x-1}\right]^{-2} = 16^x &\iff 4^{-4x+2} = 4^{2x}\\ &\iff -4x+2 = 2x\\ &\iff x = \tfrac{1}{3}\end{aligned}$$

(c) $9^x = (3^x)^2$

Solution.

$$9^x = (3^x)^2 \iff 9^x = 9^x \iff x \in \mathbb{R}.$$

(d) $\left[1 - \log(x-1)\right]^2 = 4$

Solution.

$$\begin{aligned}&\left[1 - \log(x-1)\right]^2 = 4\\ \iff\ & |1 - \log(x-1)| = 2\\ \iff\ & 1 - \log(x-1) = 2\\ & \text{or } 1 - \log(x-1) = -2\\ \iff\ & \log(x-1) = -1 \text{ or } \log(x-1) = 3\\ \iff\ & x - 1 = \frac{1}{10} \text{ or } x - 1 = 1000\\ \iff\ & x = \frac{11}{10} \text{ or } x = 1001\end{aligned}$$

4. Sea $f(x) = \pi + \dfrac{1}{x-\pi}$.

(a) Demuestre que $f(f(x)) = x$ para todo $x \in D_f$.

The function is a translation of the hyperbola $y = 1/x$ by the vector (π, π). It follows that $D_f = \mathbb{R} \setminus \pi = C_f$. Hence, if $x \in D_f$, then $f(f(x))$ exists. and

$$\begin{aligned}f(f(x)) &= \pi + \frac{1}{\left(\pi + \frac{1}{x-\pi}\right) - \pi}\\ &= \pi + \frac{1}{\left(\frac{1}{x-\pi}\right)}\\ &= \pi + (x - \pi)\\ &= x.\end{aligned}$$

(b) Demuestre que f es inyectiva.

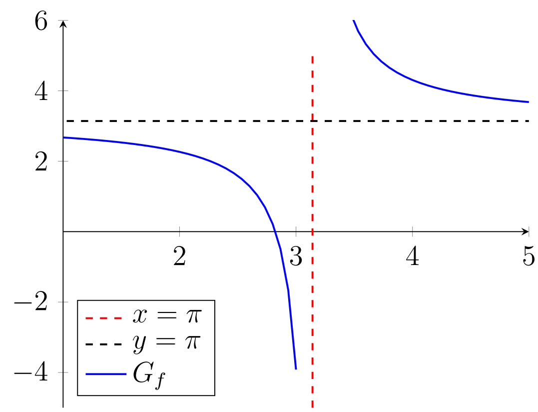
Let $x, x' \in D_f$.

$$\begin{aligned}f(x) = f(x') &\implies f(f(x)) = f(f(x'))\\ &\implies x = x'.\end{aligned}$$

(c) Halle una fórmula para $f^{-1}(y)$.

It follows from part (a) that $f = f^{-1}$. Hence $f^{-1}(y) = \pi + \dfrac{1}{y-\pi}$.

(d) Trace la gráfica de f.



5. Calcule el valor exacto de:

(a) $\sin\left(\frac{7\pi}{4}\right)$

Solution.

$$\begin{aligned}\sin\left(\frac{7\pi}{4}\right) &= \sin\left(2\pi - \frac{1\pi}{4}\right)\\ &= \sin\left(-\frac{\pi}{4}\right)\\ &= -\sin\left(\frac{\pi}{4}\right)\\ &= -\frac{\sqrt{2}}{2}\end{aligned}$$

(b) $\sin(-11\pi/4)$

Solution.

$$\begin{aligned}\sin(-11\pi/4) &= -\sin\left(\frac{11\pi}{4}\right)\\ &= -\sin\left(2\pi + \frac{3\pi}{4}\right)\\ &= -\sin\left(\frac{\pi}{4}\right)\\ &= -\frac{\sqrt{2}}{2}\end{aligned}$$

(c) $\sin(123\pi/4)$

Solution.

$$\begin{aligned}P\left(\tfrac{123\pi}{4}\right) &= P\left(30\pi + \tfrac{3\pi}{4}\right)\\ &= P\left(\tfrac{3\pi}{4}\right)\\ &= -P\left(\tfrac{3\pi}{4} - \pi\right)\\ &= -P\left(-\tfrac{\pi}{4}\right)\\ &= -(\tfrac{\sqrt{2}}{2}, -\tfrac{\sqrt{2}}{2})\\ &= (-\tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2}).\end{aligned}$$

By definition, $\sin\theta$ is the ordinate (i.e., second coordinate) of $P(\theta)$. Thus

$$\sin\left(\tfrac{123\pi}{4}\right) = \tfrac{\sqrt{2}}{2}.$$

6. (a) Halle el campo de valores de la funcion $g(\xi) = \tan\xi; \quad 0 \leq \xi \leq \pi/4$

Solution.

$CV_g = [0, 1]$

(b) Halle valores posibles de a y b si la función $f(x) = \cos x; \quad a \leq x \leq b$ tiene campo de valores $[0, \frac{\sqrt{3}}{2}$

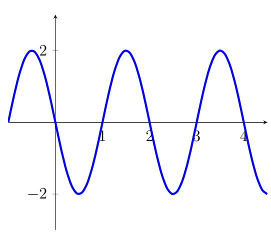
Solution.

$a = -\frac{\pi}{2}, \quad b = -\frac{\pi}{6}$

7. Sea $g(x) = 2\sin(\pi(x-1))$.

[Solution.]

(a) Trace la gráfica de g



(b) Halle el mínimo del conjunto $\{k : g(x+k) = g(x)\}$

[Solution.

The period of g is 2.]

8. Determine si las funciones siguientes son par, impar, ningunos (o ambos). Dibuje sus gráficas.

   (a) $f(x) = x^2, \quad x \geq 0$

   Solution.

   neither

   (b) $f(x) = 0, \quad x \in \mathbb{R}$

   Solution.

   both

   (c) $f(x) = 0, \quad -1 < x < 2$

   Solution.

   neither

   (d) $f(x) = \dfrac{x^3}{x^2 + x^4}, \quad x \in (-2, 0) \cup (0, 2)$

   Solution.

   odd

9. Si $\sec t = 2$ y $0 < t < \pi/2$ halle:

   (a) $\tan t$

   (b) $\sin t$

   Solution.
   $\sec t = 2 \iff \cos t = \frac{1}{2}$. Since $0 < t < \pi/2$, it follows that $t = \frac{\pi}{2}$. Hence $\tan t = 1$ and $\sin t = \frac{\sqrt{2}}{2}$

10. Sea $\alpha, \beta, \; t \in \mathbb{R}$. Demuestre:

    (a) $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

    Solution. See class notes.

    (b) $(\cos t + i \sin t)^2 = \cos 2t + i \sin 2t$