Mathematics Standards of the Puerto Rico Department of Education: Analysis and Recommendations

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Abstract

Mathematics content standards are important for establishing what every student can and should know in this field. Puerto Rico's K-12 public school mathematics content standards are contained in the Department of Education's *Estándares de Excelencia, Programa de Matemáticas* [23]. This document is analyzed with reference to a reasonable set of criteria for good standards and found wanting. In particular, we show that the standards are incomplete, highly repetitive, unfocused, replete with mathematical errors, and insufficiently demanding of the students. The philosophy of the PR standards is also examined and a case made for neutrality of content standards with respect to educational philosophy. Our analysis indicates a pressing need for the development of new mathematics standards for the public schools of Puerto Rico and a greater participation of practising mathematicians (as well as educators) in this process. Part 4 examines briefly some alternative sets of standards which are currently being tested in Puerto Rican schools. In Part 5 we look at some lessons that can be drawn from the Third International Mathematics and Science Survey. An examination of the 2002 curriculum reform and its likely impact on mathematics education is presented in Part 6. Since an appropriate reading level is a necessary condition for successful mathematics education, the latest research in reading instruction is examined in Part 7. This research provides overwhelming evidence that methods of reading-writing instruction which exclude explicit phonics training are inferior. Studies demonstrate that poor reading ability is strongly linked to attrition in the schools as well as delinquency. In view of the current high attrition rates, these results are particularly relevant to Puerto Rico. We end with a comprehensive list of recommendations.

Contents

1 Introduction

Mathematics content standards for the K-12 public schools of Puerto Rico are presented in the document, *Estándares de Excelencia–Programa de Matemáticas*, Departamento de Educación, Gobierno de Puerto Rico, Agosto 2000 [23], henceforth referred to as the **PR-DOE Standards**. According to page v. of the **PR-DOE Standards**

> *Los maestros utilizan los estándares para identificar aquello que deben enseñar en cada grado y nivel, con qué fines enseñar y qué esperar de los estudiantes como resultado de lo enseñado. De esta manera, no tendrán que decidir aisladamente qué es suficiente o cuánto es suficiente en el proceso educativo.*

Purportedly, a main aim of the **PR-DOE Standards** is to specify content. In section 2 we consider a reasonable set of criteria for good mathematics standards and in section 3 we examine the extent to which the **PR-DOE Standards** meet these various criteria.

2 Characteristics of Good Standards

Educational standards typically comprise a series of statements describing what students should know and be able to do. Standards should specify the knowledge and skills that schools are expected to teach and students are expected to learn. They should also provide a description of what students should know after a particular grade or course is completed. Standards should also include mechanisms for evaluation. Quirk [67] has listed desirable characteristics of good K-12 mathematics standards. These criteria are reasonable and provide a benchmark for analysis. The failure of a standard to meet a particular criterion suggests the need for serious discussion and possible remediative action.

2.1 Characteristics of Individual Mathematics Standards

According to Quirk [67], a mathematician, educator and professional course developer, individual mathematics standards should have the following characteristics:

- **Focused:** Each standard should cover exactly one math topic, where a math topic is a small closely related set of math facts and math skills.
  - A math topic is a conceptual "chapter" of math knowledge, not a "book".
  - The time to learn a math topic is measured in days or weeks, not years.
  - Example: "Find the equation of a straight line given two points on the line".
- **Specific:** Each standard should be stated in the most explicit possible way.
  - Different K–12 math teachers should easily arrive at the same understanding of the standard.
  - Example: "The student will memorize the 25 multiplication facts, from $1 \times 1 = 1$ through $5 \times 5 = 25$".
- **Basic:** Each standard deals with a core knowledge math topic.
  - Math needed for everyday life.
  - Math needed to develop logical and abstract thinking skills.
  - Foundational prerequisite math needed to learn more advanced math.
  - Math needed to acquire knowledge in other subject areas that utilize math.
- **Teachable:** Is it possible to teach the topic in a step-by-step manner?
  - To the extent that a mathematics topic is vaguely described or unfocused it is correspondingly difficult to teach.
- **Measurable:** Student mastery can be easily evaluated by an objective test.
- **Linked to Grade:** Each standard should link to exactly one K–12 grade or to a specific named course.
  - One teacher should be responsible for teaching the standard, measuring student mastery, and taking timely corrective action when mastery is not achieved.
  - If a standard is properly focused, the time needed to master the math topic should never exceed a few weeks. If years are required, the standard is too broad.

  - A standard may extend a standard for an earlier grade, but it should not be identical to a standard for an earlier grade or simply rephrase a standard for an earlier grade.

- **Concise:** Standards should be stated using the minimum number of words needed for clarity.

- **Not Redundant:** Different standards should deal with different math topics.

- **Genuine Math:** Each standard should describe mathematics in the sense recognized by a **practising** mathematician.

  - Unfortunately, as we will show later, much recent "reform" mathematics is no longer recognizable as mathematics.

2.2 Characteristics of a Complete Set of Math Standards

According to Quirk [67], a complete set of mathematics standards should have the following characteristics:

- **Brief:** The K–12 math standards document should consist of:

  - 30–50 pages
  - 2–4 pages of standards per grade or course
  - 15–30 standards per grade or course

- **Selective:** A very small percentage of known mathematics can reasonably be covered during the K–12 years.

  - The standards should collectively identify an essential core of mathematics which all students should learn. This does not preclude teachers from doing more. To the extent that time and circumstances permit, they should do more.

- **Coherent Structure:** Each standard should be properly sequenced after all necessary prerequisite standards.

  - The logical structure of a set of standards should be compatible with the structured nature of mathematics. New mathematics knowledge is built on previously learned mathematics knowledge.

- **Pedagogically Neutral:** The standards should describe the required mathematics content only. They should **not** specify teaching methods. They should not discuss teaching philosophy.

A good example of a set of standards which adheres to these attributes are the *Mathematics Content Standards for California Public Schools, Kindergarten through Grade Twelve* [9] which were adopted by the California State Board of Education in 1997.

In the following section we analyze the extent to which the **PR-DOE Standards** meet these criteria.

3 The PR-DOE Standards

3.1 Concerning the Format

A number of elementary and high school teachers with whom I have spoken have complained that the format of the **PR-DOE Standards** is so confusing and difficult to follow that they do not read them. Accordingly, we start our analysis with a look at this important aspect. Reasons for the teachers' confusion are not hard to find. Information relating to a given grade is dispersed over ten different sets of standards; namely, 5 sets called *Standards of Excellence* and 5 sets of *Process Standards.* The 5 sets of Standards of Excellence are called *Number and Operation, Algebra, Geometry, Measure, Data Analysis and Probability.* For each standard of excellence, there are 4 separate tables corresponding respectively to grades K-3, 4-6, 7-9, and 10-12 but not in a correct order. For example, the geometry standards are given in the order: grades 10-12, 4-6, 7-9, and K-3. Each of the tables has three columns: *Content*, *Standards of Execution*, and *Standards of Assessment.* The contents of the *Content* column are identical in each of the tables K-3, 4-6, 7-9, and 10-12. The column *Standards of Assessment* is the most confusing. It possesses three "sub sections" labelled *Diagnostic*, *Formative Evaluation*, and *Summative Evaluation.* The subsection *Diagnostic* makes reference to courses Matemática Integrada I and II, Álgebra, and Geometría and some others, to which there are no bibliographic references anywhere in the rest of the document. Thus, despite containing over 250 pages, the **PR-DOE Standards** document is not self contained.[1] Below the names of each of the aforementioned courses appear lists of expectations which are identical to items in the *Standards of Execution* column except that verbs are rewritten in the infinitive. For example, in the column *Standards of Execution*, page 21, we read,

Identifica patrones pertinentes en el contexto de su vivir cotidiano.

In *Standards of Assessment* on page 21 we find:

Identificar patrones pertinentes en el contexto de su vivir cotidiano.

Sometimes, items which appear in the column *Standards of Assessment* do not appear in the other two columns. For example on page 34, we see in the column *Standards of Assessment*

Aplicar el Teorema de Pitágoras en la solución de problemas

However, there is no specific mention of this theorem in the *Content* and *Standards of Execution* columns. How can students "apply" the theorem if they have not previously studied it?

To summarize, the **PR-DOE Standards** are not self contained; they are poorly formatted and do not present material in a logical manner. The user is unable to easily find the information sought. The inordinate length of the document detracts from its convenience and adds to the costs of reproduction and distribution.

3.2 Concerning Content

3.2.1 Some Serious Omissions

A large number of important mathematical concepts and results are not mentioned in the **PR-DOE Standards**. We now consider a few of the more serious omissions.

Explicit study of the concept of *definition* –critical to all discourse mathematical or otherwise– is omitted from the standards. Since it is all but impossible to locate a current high school math textbook which actually **discusses** the concept of definition, we remind the reader that according

[1] A visit to the Department of Education in search of more information yielded only documents labelled draft or working document, some from as far back as ten years. One is left with the impression that these documents will disappear into oblivion before losing their draft status.

to Aristotle, 384-322 BC, a species is defined by giving its *genus* and its *differentia.* The differentia specifies the attributes which characterize the species within its genus. Mathematical definition follows the same pattern.

Example. An integer n is *even* if there exists an integer k such that $n = 2k$.

In this example, we define the term *even* (the species). It is convention to place the species to be defined in italics or to underline it. The role of genus is played by the set of integers. The differentia is expressed by the predicate, "there exists an integer k such that $n = 2k$". This predicate specifies the subset of the set of integers which we call even.

That the concept of definition is not obvious is attested to by the fact that most students (and many teachers) have difficulties when asked to give a correct definition. The authors of the **PR-DOE Standards** write in a way which suggests a lack of understanding of the concept. On page 16 we read,

El estudiante comprende un entendimiento [sic] *de números grandes y pequeños en representaciones variadas de ellos.*

It is not clear what is intended here by *large* number since there are no general mathematical definitions of *large* and *small.* These are relative concepts. The statement on page 11 of the **PR-DOE Standards**

El estudiante es capaz de identificar números impares

is a poor substitute for *know and understand the definition of odd number.* The only way students can prove simple statements, such as *the sum of two odd numbers is even*, is if they know and understand the definitions involved. Anything else is not mathematics. A student who does not know a correct definition of a concept cannot begin to claim understanding. Thus, it is essential that the concept of definition be **explicitly** taught.

The word *axiom* appears nowhere in the **PR-DOE Standards**. In particular, in the geometry standards, no guidance is given as to which statements should be taken as axioms. For example, should the congruency condition *SSS* be proven or assumed for simplicity? The specification of a set of axioms for geometry is important since the choice is far from unique. Too many axioms leads to vacuity, too few to unwarranted technical difficulty. Axioms are not self evident truths. In non Euclidean geometries, the negation of the parallel postulate is an axiom and therefore true, but it is **not** a "self evident" truth in a locally Euclidean world such as the one we inhabit.

The notion of *primitive* concept is absent from the standards. What are the students expected to make of the notions of point and line when they study Euclidean geometry? The more astute student will see immediately that a statement such as "a set is a collection of objects" is not a good definition and should be made aware that the concept of *set* is primitive.

Reasoning and mathematical proof are listed as process skills and given in separate process standards when in reality they are inseparable from mathematics. Mathematical reasoning cannot be mastered in isolation from mathematical content. Similar remarks apply to the other process skills. Many of the so called process skills are vaguely defined and more sociological than mathematical.

There is no clear indication anywhere in the **PR-DOE Standards** that the **proof** of the very important and basic Pythagorean theorem is to be covered. Needless to say, the converse of this theorem is nowhere mentioned. All the evidence suggests that this theorem is not well taught in the schools. In fact, I regularly encounter students in my classes at the University of Puerto Rico who claim never to have seen a proof of this classic theorem -and worse, many inform me that they have never seen a proof of **any** mathematical result in their school career. If true, this constitutes educational fraud. The failure to teach basic geometry properly in the high school limits a student's access to scientific careers such as chemistry as well as technical careers such as carpentry for which good geometric training is highly desirable.

Concerning the fundamental notion of *function*, the *Content Standards* (page 21) state that the student should be capable of *"understanding patterns, relations and functions."* [this and succeeding translations provided by the author] The first mention of the word function in the corresponding *Standards of Execution* and *Assessment* is in grades 7-9 (page 26) where we find:

El estudiante identifica funciones en lineales [sic] *o no lineales y las contrasta con tablas, gráficas (plana o al relieve) y/o ecuaciones.* [sic]

Then on page 29 we read:

El estudiante selecciona, convierte y utiliza diferentes representaciones y patrones para comprender y generalizar funciones explícitamente definidas, recurrentes y relaciones. [sic]

In order to be able to identify functions in tables and graphs one needs to have carefully studied the definition of the concept at some point. However, it is not clear from the standards where this point occurs if at all. A teacher reading the above could easily assume that all that is required is an informal discussion. It is important for the teacher to know whether and when the students are supposed to learn the definition of a function. Moreover, phrases such as "comprender y generalizar funciones" are very vague and different teachers could easily arrive at different interpretations.

Many other fundamental theorems such as the division algorithm and Euclid's algorithm are omitted from the **PR-DOE Standards**. The standard algorithm for division depends on the division algorithm. Terminating and repeating decimals cannot be understood without reference to the remainder term in this theorem. The division algorithm also generalizes to polynomials. At a slightly more advanced level it is used to obtain the important mini-max characterization of the greatest common factor. Actually, it is not entirely clear from the **PR-DOE Standards** when or if the **standard** algorithms for long multiplication and long division are to be learned. The best that can be discerned (see page 11) is that by some unspecified time between the fourth and sixth grades, the student is expected to

utilizar las operaciones básicas con números cardinales haciendo uso del papel y lápiz o calculadora de acuerdo con sus habilidades individuales.

The wording here is sufficiently vague that some teachers could assume that students are expected to "construct" their own ad hoc strategies for multiplication and division, a practice encouraged by the widely criticized National Council of Teachers of Mathematics (NCTM) 1989 Standards [55] or, worse, use only calculators. It should be the aim of the schools that all students know how to do the **standard** algorithms. As Wu [84] points out,

deep understanding of mathematics ultimately lies within these skills.

Thus, the essential vocabulary items of mathematics: definition, axiom, primitive, etc., are at best only implicit in the standards. How, therefore, can students meet the requirement of page 104 of the **PR-DOE Standards** that they be able to *use mathematical language to express precise mathematical ideas?* Furthermore, we learn that much of what is actually required of the student is **not mathematical but sociological** in nature. The *Standards of Execution* require that the student:

- *Communicate effectively with his peers.* (**PR-DOE Standards**, page 103).
- *Criticize, self criticize and accept suggestions to better his mathematical thought and* **quality of life**. [emphasis added] (**PR-DOE Standards**, page 104).
- *Adapt and increase his mathematical knowledge and achieve integration of his sociocultural surroundings using bicultural and bilingual resources.* (**PR-DOE Standards**, page 98 and repeated on pages 100, 102, 104.)

3.2.2 Inductive versus Deductive Reasoning

The **PR-DOE Standards** do not use the word *proof* in the manner acceptable to mathematicians. In mathematics only deductive argument constitutes a proof. However, according to page 93, the student in grades 10-12 is required to

> *Prove, demonstrate and justify according to a variety of strategies.*

These include

> *Reason and prove coherent and logical conclusions, both oral and written, using tables, paragraphs and technological equipment.*

Thus, the **PR-DOE Standards** endorse "informal" proofs **for all** K–12 years. Informal reasoning is not mathematics and too much reliance on such reasoning into the later grades contradicts the main reason for studying mathematics; namely, the development of the ability for abstract thinking. According to Allen [1],

> *Mathematics is EXACT, ABSTRACT and LOGICALLY STRUCTURED. These are the ESSENTIAL and CHARACTERIZING properties of mathematics which enable it, WHEN PROPERLY TAUGHT to make unique and indispensable contributions to the education of all youth.*

At the same time as de-emphasizing deductive reasoning, the **PR-DOE Standards** place an excessive emphasis on the recognition of patterns. For example, in the *Standards of Execution* on page 21 we read that the student should:

- *Reconoce, lee, describe y amplía patrones repetitivos y crecientes.*
- *Describe, extiende y hace generalizaciones sobre patrones numéricos y geométricos.*
- *Identifica patrones pertinentes en el contexto de su vivir cotidiano.*
- *Identifica y amplía un patrón con objetos concretos, siluetas,...*
- *Resuelve patrones numéricos utilizando estrategias de conteo.*

In the corresponding *Standards of Assessment* we find:

- *Imitar patrones de ritmo y movimiento.*
- *Identificar patrones de colores con dos elementos.*
- *Identificar patrones pertinentes en el contexto de su vivir cotidiano.*
- *Completar patrones numéricos utilizando estrategias de conteo 2 en 2, etc.*
- *Extender un patrón con objetos concretos y siluetas.*
- *Identificar patrones numéricos y geométricos.*
- *Extender un patrón numérico.*
- *Representar y analizar patrones y relaciones usando lenguaje matemático.*
- *Explorar patrones y relaciones mediante modelos concretos.*

This seems to derive from the pervasive influence of the questionable "standards" of the NCTM [55] present in the **PR-DOE Standards**. As Quirk [67] has pointed out

Pattern recognition is an inductive rather than a deductive process and like inductive reasoning, is more properly associated with science, than mathematics... Recognizing patterns depends on remembered math facts. Recognizing the pattern in the sequence (5, 10, 17, 26, 37, 50, . . .) requires a knowledge of perfect squares. Recognizing the pattern in (10, 12, 16, 18, 22, 28. . .) requires a knowledge of prime numbers. Pattern recognition is not directly teachable, but pattern-recognition skills improve as a by-product of learning math facts. Thus, if pattern-recognition skills is the goal, teach genuine math facts!

On the other hand, *mathematical induction* as opposed to inductive reasoning is completely omitted. Without mathematical induction a student cannot prove many important facts such as *every integer is either even or odd* or that $n + m = m + n$ or that $n < 2^n$. Mathematical induction is needed in applied disciplines such as chemistry. For example, to obtain the general formula C_nH_{2n+2} for the saturated hydrocarbons one typically uses induction. The principle of mathematical induction is a basic and central idea of mathematics and should be included in the high school curriculum.

3.2.3 Concerning the Use of Calculators

A major cause for concern is the emphasis the **PR-DOE Standards** place on the use of calculators in the early grades (K-3, page 9). This is not in accord with best practice. According to Williamson Evers [28], a research fellow at the Hoover Institute and specialist in educational policy,

In the elementary school grades, these devices serve no constructive purpose in assessing knowledge of mathematics, and their abuse can cause considerable damage to the education of children. They allow students to compute with numerical values having a large number of significant digits, but this only obscures basic mathematical principles, which the use of simpler numbers reveals. Calculators and computers may actually inhibit accurate assessment since these devices encourage overuse of the primitive problem-solving strategy of "guess and check" and thus discourage use of mathematical techniques of problem solving.

It is essential that students have complete facility with all of the arithmetic operations: addition, subtraction, multiplication, and division. These operations arise again in the context of algebra, for example in arithmetic combinations of polynomials, and later in calculus, when the need to manipulate power series arises. Nothing is gained by requiring students to compute by pushing buttons, especially in elementary school, and much can be lost.

The top scoring nations in 4th grade mathematics, according to the TIMSS report, do not allow the use of calculators. The report states: "In six of the seven nations that outscore the U.S. in mathematics, teachers of 85 percent or more of the students report that students never use calculators in class".

As we discuss in Section 5, the most mathematically successful nations **do not allow** the use of calculators before pencil and paper calculations have been mastered and do not allow the latter until mental proficiency has been achieved.

3.2.4 Concerning the Level

The general mathematical level of the **PR-DOE Standards** is well below international norms. For example, in the California Board of Education Standards for grade seven [9] we read

Know and understand the Pythagorean theorem **and its converse** *and use it to find the length of the missing side of a right triangle.* [emphasis added]

The corresponding topic is not covered until grades 10-12 of the **PR-DOE Standards** which merely require that students *apply the Pythagorean theorem to the solution of problems.* Moreover, as mentioned previously, the converse of this important theorem is not even mentioned. Actually, the Japanese textbook *Mathematics 1, Japanese Grade 10* [41], studied by 97% of all Japanese students, covers more material and with greater level of rigor than do the **PR-DOE Standards** for grades 10-12.

Given that the **PR-DOE Standards** have very low expectations of the student, it is not surprising that the majority of students entering the University of Puerto Rico are ill prepared for college-level courses in fields such as biology, chemistry, architecture, nursing, and, indeed, any field which requires analytical thinking. This is evidenced by the fact that remedial mathematics is now the mathematics course with the highest enrollment. The general low level of mathematics in the high schools is also reflected in the number of university students majoring in mathematics. UPR Río Piedras, a campus of over 20,000 students, produces less than 5 math majors a year, whereas universities in other countries typically have hundreds of math majors. For instance, at Bryn Mawr college in the U.S., 11 percent (35 out of a class of 316) of bachelor degrees awarded in 2002 were in Mathematics. At the University of Warwick in England there are over 700 undergraduate mathematics majors out of a total undergraduate population of just over 8,000. In view of the importance of a good level of math training to the general economy, these statistics should be of great concern if Puerto Rico is to successfully compete with other nations.

3.3 Concerning the Appendices

The **PR-DOE Standards** contain approximately 130 pages of miscellaneous appendices with neither index or page numbers. The general index at the beginning of the document gives no indication as to the content or purpose of the appendices.

Appendices 1 and 2 concern, respectively, the evaluation of handicapped students and students whose first language is not Spanish. Since none of the information is specific to mathematics, this large section of 36 pages would be much better placed in an independent document and simply cited as needed.

Appendix 3 is a description of different methods of assessment, supposedly "adapted from" the book *"Assessment" ... de la teoría a la Práctica* [sic] which is not correctly cited; the name of the author, the publisher, and date being omitted. Apart from being inefficient, the assessment methods (comic strips, poems, reflexive diaries, etc.) are in general very subjective, and hence inadequate for summative evaluations of the students' performance (see Hirsch [35]). Besides, none of these methods is specific to mathematics, so there is no good reason to include them within a **mathematics** standards document. Accordingly, we recommend that this appendix be deleted.

Appendix 4 is entitled "Destrezas Prueba Diagnóstica K-12". This section is merely a repetition (with a different title!) of the *Standards of Assessment* given in column 3 of the tables on pages 7 thru 74. Thus, the only outcome of this 12 page appendix is to further confuse the already bewildered reader.

Appendix 5, entitled "Matemática Discreta", starts with an unintelligible "definition":

Matemática Discreta: rama de las matemáticas que estudia objetos o ideas que pueden dividirse en partes separadas o discontinuas. Difiere de la noción de continuidad en la cual se basa el álgebra y el cálculo. [sic]

It continues:

Se clasifica en tres categorías:

1. *Problemas de existencia: estudio de si un problema dado tiene solución.*

2. *Problemas de conteo: Investiga cuántas soluciones pueden existir para problemas que tienen solución.*

3. *Problemas de optimización: hace énfasis en la mejor solución para un problema dado.*

I asked two professional mathematicians to identify the branch of mathematics to which items 1 and 3 refer. Neither had the remotest idea. As regards item 2, *Investiga cuántas soluciones pueden existir para problemas que tienen solución*, is a general statement relevant to **all** branches of mathematics and is not specific to the field of *"conteo"*, let alone Discrete Mathematics. Therefore, the above is no more than a caricature of Discrete Mathematics.

Appendix 5 ends with a list of so called examples and applications of Discrete Mathematics such as *Técnicas de conteo (combinaciones de ropa de una persona)*, hardly the most inspiring application. The list given is far too general to be of any guidance to the teacher. Many of the areas (graph theory, algorithm theory, etc.) mentioned are in themselves mathematical oceans. The application to quality control is traditionally classified under statistics. Topics such as logic and set theory are general foundational material. The most mysterious of this strange appendix is that there is no mention of Discrete Mathematics or Discrete Mathematics courses in the rest of the document. It appears that this appendix –like the human biological appendix– is purely vestigial in nature and hence removable. In reality, Discrete Mathematics contains much core material that is essential for all students. However, a curriculum which is merely a caricature will be of no help to the teacher in deciding between what is important and relevant and what is inessential.

Appendix 6 of the **PR-DOE Standards** contains a random selection of student activities. They follow no particular theme and are uncorrelated with the standards. A teacher's manual might be a more appropriate place for such material. Unfortunately, the mathematical quality of the activities ranges from low to nil (see, for example, the activity entitled *Los Cuadrados*). Moreover, some of the examples are sociological and behavioral in nature. In section 3.4 we will examine the mathematical content of a number of these activities.

Therefore, some of the appendices should be removed from the **PR-DOE Standards** because they are repetitious or belong somewhere else; others because they are mathematically deficient. A minimum expectation of a set of mathematics standards is that they be mathematically correct. As Wu [86] has pointed out in reference to a set of (rejected) California standards,

> *Too often mathematics educators and administrators lose touch with mathematics... The large number of mathematical errors in the Commission's standards also point to an intellectual problem far removed from the political fray. As the errors begin to pile up, they send out the unmistakable message that these standards were written by people whose mathematical understanding is inadequate for the task, and whose vision is therefore unreliable as a guide to lead students of California to a higher level of mathematical achievement. Such being the case, the so-called "conceptual understanding" embedded in this document is thus of questionable value at best.*

3.4 Concerning the Sample Activities

Appendix 6 of the **PR-DOE Standards** provides a –somewhat random– selection of student activities. As mentioned earlier, they follow no particular theme and are uncorrelated with the standards. In this section we examine some of the activities.

3.4.1 Activity: Jugando con el Transportador

In this activity for sixth graders we find the following instructions:

1. Every group will prepare 6 different angles on the construction paper.

(a) Forming two rays with rice and glue

(b) Forming two rays with straws and glue

(c) Forming two rays with string and glue

The students are then instructed to measure the angles using a protractor. Historically, the ruler and compass have proved to be the "manipulatives" par excellence for drawing angles; with them, many wonderful constructions are possible. Moreover, they are cheap and widely available. Why then are Puerto Rican children reduced to using time consuming glue and string instead? In the evaluation section for this activity, entitled *Hoja de Cotejo*, the teacher is asked to evaluate the student on eight criteria. The first six are actually sociological in nature:

Discute el problema con sus compañeros,
Comparte materiales con los compañeros,
Ayuda a otro compañero sin indicarle la solución, etc.

The seventh is *puede construir correctamente los ángulos*. Only the **last**, *Can measure angles correctly using a protractor*, is remotely connected to mathematics. What are the consequences of this evaluation for students who fail to cooperate? A child may have good reasons (bullying, etc.) not to discuss the problem with his classmates.

This activity is a typical example of psychological assessment masquerading as academic testing. This has serious implications concerning privacy, particularly given the growing tendency to computerize student data and link together databases. Already, (see, for example, Eakman [21], [22], and Sykes [77]), many neurologically normal "uncooperative" children have been labelled as suffering from vaguely defined and elusive "diseases" such as "attention deficit disorder", "dyscalculia", "sequencing ability defects", or in some other way learning disabled, on the basis of psychological assessments by overzealous teachers and counselors –unqualified as psychiatrists. Such data, when entered into electronic records, can potentially affect a student's chances of college admission or employment prospects. Students are expected to construct their mathematical reality but their social one is imposed by the teacher. Evaluation should focus on a child's mathematical competencies.

The level of this activity is also of concern. While Puerto Rican students in grade 6 are still playing with educational training wheels such as rice and glue, in a typical sixth grade Russian text [59] we find questions and activities such as the following:

1. *Draw a circle of any radius. Mark on the circumference points A, B, and C such that the segment AB forms the diameter of the circle. Draw segments AC and BC. Measure angle* ACB.

2. *Calculate and draw the following angles: 1) 30% of 180; 2) 60% of 70; 3) 45% of 160.*

3. *On a 6th-grade math test 25% of the class earned an A, 35% a B, 30% a C, and 10% a D. Draw a bar graph depicting the grade distribution.*

4. *The center of a circle is the point O, the length of its radius r. Where is the point P located if: a)* $OP < r$*; b)* $OP = r$*; and c)* $OP > r$*?*

5. *A semicircle has been cut out of a rectangle (fig. 4.13). Make the necessary measurements and calculate the perimeter of the remaining solid.*

6. *In his science class, Andy built a model plane and took it out for a test run. If the plane flew a circle with a 60m. radius in 1.8 seconds, at what speed was the plane flying?*

7. *The famous Greek mathematician Archimedes determined that* $310/71 < \pi < 31/7$. *Compare the circumference of a circle using 310/71 and 31/7 as* π *if the circle's radius is 497 cm.*

It is clear from these examples that the Russian sixth grade has considerably higher mathematical expectations of the students than do the **PR-DOE Standards**. The topics covered here (bar graphs, percentages, angle, approximations, etc.) are basic and should be known by **all** students before they enter high school. The failure of the educational establishment in Puerto Rico to instill these topics in students helps explain why large numbers of university students require remediation in sixth grade mathematics, –now euphemistically called *quantitative skills* [62]. The economic and emotional cost of all this remedial activity is huge.

3.4.2 Activity: Comparamos Nuestra Altura

This tenth (!) grade activity claims to be designed for the course *Matemática Integrada I*. There are 3 activity sheets. According to sheet I, the students are divided into groups of 5 and asked to:

1. Predict a relation between the length of a person's forearm and height.

2. Measure the lengths of forearm and height for students in their group and graph the results.

3. Decide whether the graph supports the formula predicted in 1.

4. Use their graphs to make a prediction of the length of forearm given a height of 171 cm.

5. Construct a dispersion diagram using a graphics calculator.

The mathematical purpose of this activity is not clear. It could be that a linear regression is intended, and that by the term "graph" in item 4 is meant the regression line. However, this is nowhere stated. If this is the intention, there should be a discussion of the mathematical principles involved (the method of least squares, the coefficient of linear correlation, etc.). If, on the other hand, the students are expected to "eyeball" a rough straight line through the data points, then there is almost no discernible mathematical benefit from all this activity. In item 5, it is not clear what the author means by dispersion diagram or for what purpose it is intended. Perhaps the author is referring to a residual plot in a linear regression. A glance at the draft DOE text for the course Matemática Integrada I sheds no light on these matters. It contains no discussion of regression, correlation, or of dispersion diagrams. Activity sheet II is ambiguous for similar reasons. It asks students to use the "graph" which they "constructed" in activity I to explain the meaning of a point located above the "line", and to provide examples which support their answers. The rubric for evaluating this activity is equally bizarre. For example, one point is given for explaining with three **or more** errors the relation between points and the "line" drawn. Activity sheet III asks students to complete the following phrase

la calculadora gráfica fue beneficiosa porque ...

This is begging the question. Large scale studies have failed to find significant educational benefits of using calculators (or computers) in education (see Oppenheimer [60]; Koblitz [40]) yet the students are required to **assume** that they are beneficial. In reality, as pointed out earlier, use of computers and calculators can be harmful in that they can obscure underlying mathematical principles. If students were taught to do a detailed regression analysis by hand they would (a) Understand the calculations and (b) Immediately see the value of using a machine for large data sets.

3.4.3 Activity: Animales en la Finca

In this activity for kindergarten we read

El estudiante completará una gráfica de barras donde mostrará la cantidad de animales que hay en una lámina provista.

The activity described is perfectly reasonable for the grade level. However, the coverage of the topic of bar graphs in later grades is symptomatic of a lack of balance. In Appendix 4, *Destrezas Prueba Diagnóstica*, we find

Third Grade: *Interpretar gráficas de barra.*
Fourth Grade: *Interpretar gráficas de barra.*
Sixth Grade: *Interpretar información en una gráfica de barra.*
Eighth Grade: *Interpretar gráficas de barra.*

Then, as if this were not enough, we find in Appendix 6, an activity on bar graphs for the ninth grade entitled *Empleo y Tendencias* in which we read:

2 out of 4 points will be given for completing a bar graph with 3 or more errors.

Since there exist arbitrarily large numbers greater than 3 the graph can be infinitely wrong yet still score 50%. It does not take a rocket scientist to understand why students drop out of school. *No drill and kill* is one of the mantras of constructivist education yet here is an example of an essentially trivial topic repeated ad nauseam over at least 9 years. Moreover, syllabi indicate that most students will see this topic again in the university. After remedial mathematics courses, statistics is the most widely taught course in universities. There is too much emphasis in the **PR-DOE Standards** on non fundamental material such as elementary statistics and exploratory data analysis. For a student well trained in elementary foundational mathematics, this type of activity is trivial.

3.5 Concerning Recommended Textbooks

Choice of textbooks is of great importance both for students and for teachers. A deficient mathematics textbook can cause incredible confusion and lead to serious misconceptions. In fact, one of the major findings of the Third International Mathematics and Science Survey (see Section 5) was that U.S. school mathematics textbooks are markedly inferior to those used in the mathematically top performing nations.

Although the **PR-DOE Standards** do not explicitly recommend textbooks, some of the courses referred to therein have teacher's guides which do. For example, on page 7 of the (Draft[2]) Teacher's Guide [24] for the course Integrated Mathematics II, it is stated that

teachers should refer to the recommended textbooks to find exercises and problems which correspond to the skills enumerated.

The list of recommended texts raises a number of issues concerning both the quality of the texts and the ethics of the recommendation process. A co-author of one of the "recommended" textbooks, *Estadística Descriptiva*, was a member of the very same panel who wrote the Teacher's Guide in which the book is recommended. Notwithstanding the book's merits or lack thereof this is a clear conflict of interest. The current practice of consistently omitting the authors' names of recommended books opens the door to this kind of situation.

As to the quality of the recommended books, a review [12] of Cathy L. Seeley, Barbara Alcalá, Penelope P. Booth, et al., *Secondary Math: Foundations of Algebra and Geometry* (Scott Foresman/Addison-Wesley, 1996) concludes that

[2]See footnote 2.

The book is so deficient in content that it should never be considered for use in introductory algebra.

Another of the recommended textbooks, *Focus on Algebra* by Randall Charles, Alba González Thompson, Trudi Hammel Garland, et al. (Scott Foresman/Addison-Wesley, 1996), received a scathing review by Marianne M. Jennings in a widely quoted article in the *Wall Street Journal* entitled *MTV Math Doesn't Add Up* [39]. This textbook has been widely dubbed with the unfortunate title "Rain forest Algebra" and has generated a huge controversy throughout the United States. Articles ridiculing it have appeared in many major newspapers and magazines. The book has even been criticized in the United States Congress [8]. According to Senator Byrd in a widely quoted speech

*... By the time we get around to defining an algebraic expression we are on page 107. But it isn't long before we are off that boring topic to an illuminating testimony by Dave Sanfilippo, a driver with the United Parcel Service. Sanfilippo tells us that he "didn't do well in high school mathematics $\cdots$ but that he is doing well at his job now because he enters * * * information on a pocket computer"– hardly inspirational stuff for a kid struggling with algebra.*

From there we hurry on to lectures on endangered species, a discussion of air pollution, facts about the Dogon people of West Africa, chili recipes and a discussion of varieties of hot peppers –no wonder our pages are having difficulty containing themselves. They are almost in stitches– what role zoos should play in today's society, and the dubious art of making shape images of animals on a bedroom wall, only reaching a discussion of the Pythagorean Theorem on page 502.

He goes on to remark that

This awful textbook obviously fails to do in 812 pages what comparable Japanese textbooks do so well in 200. The average standardized math score in Japan is 80. In the United States it is 52.

The Teacher's Manual for this text is equally unsound. For example, Richard Askey [3], a University of Wisconsin mathematician, considers the following example (problem 7 on page 736):

Explain why $4^{1/2}$ is rational while $5^{1/2}$ is irrational.

The answer given in the Teacher's Manual is:

$4^{1/2}$ *is rational since it equals* 2. $5^{1/2}$, *in its decimal form, does not terminate or repeat* [sic] *and therefore cannot be written as an integer over an integer.*

Professor Askey is currently offering

$100 for a proof of this last claim, i.e., for a proof that $5^{1/2}$ has a decimal expansion which does not repeat, without first showing that it is irrational.

Because of the public outcry surrounding *Focus on Algebra*, the public schools in many jurisdictions have long discontinued the use of this book. So why, five years after all of this controversy, is this a recommended text in a guide for teachers in Puerto Rico?!!!

The above examples serve to indicate a serious need for an examination of the PR-DOE's policy for textbook recommendation and adoption. Are experts (including practising professional mathematicians) being consulted before books are adopted or recommended? Finally, it is important that a mathematical standards document not advocate a particular methodology or a particular book. The purpose of standards is to specify **what** should be taught at each level and not **how** to teach it. As regards books, they are continually changing and new editions written, so they should not be enshrined in standards documents which may be infrequently revised.

3.6 Concerning the Philosophy

The **PR-DOE Standards** are not ideologically neutral but, as clearly stated on page i., based on the constructivist educational paradigm. The Department of Education claims that the standards take into account philosophical, psychological and sociological principles. Moreover, it claims that

> *Estos fundamentos de la educación se pondrán de manifiesto conjuntamente con los principios pedagógicos que han probado su efectividad en armonía con las nuevas tendencias respaldadas por la investigación* [sic].

Renowned psychologists John Anderson, Lynne Reder and Herbert Simon –the latter a Nobel Laureate– have carried out a systematic study of research in the application of cognitive psychology to mathematics education. A summary of their findings can be found in their article "Applications and Misapplications of Cognitive Psychology to Mathematics Education" [2]. The results are not supportive of the constructivist's notion that students "construct" their own knowledge. Nor do they offer support for the idea that skills can best be learned in a specific (i.e., real world) context. According to Anderson, Reder, and Simon,

> *"It is not the case that learning is totally tied to a specific context. [...] In fact, there are many demonstrations of learning that transfers across contexts and of failures to find any context specificity in the learning." "Knowledge does not have to be taught in the precise context in which it will be used, and grave inefficiencies in transfer can result from tying knowledge too tightly to specific, narrow contexts." "Representation and degree of practice are critical for determining the transfer from one task to another, and transfer varies from one domain to another as a function of the number of symbolic components that are shared." "What is important is what cognitive processes a problem evokes and not what real-world trappings it might have." "When students cannot construct the knowledge for themselves, they need some instruction. There is very little positive evidence for discovery learning and it is often inferior. In particular, it may be costly in time, and when the search is lengthy or unsuccessful, motivation commonly flags." "To argue for radical constructivism seems to us to engender deep contradictions. Radical constructivists cannot argue for any particular agenda if they deny a consensus as to values. The very act of arguing for a position is to engage in a value-loaded instructional behavior. It would seem that radical constructivists should present us with data about the consequences of various educational alternatives and allow us to construct our own interpretations. (But data beyond anecdotes are rare in such constructivist writings.)"*

Other serious criticisms of constructivism have been made by Grossen [33], Kozloff [45], Matthews [53], Carnine [10], Thomas [79], and Jennings [39]. Although some constructivist ideas are true (for example, that students are not just passive learners and that they have an interest in understanding themselves and their world), unquestioningly accepting all of its tenets is unwarranted[3].

Piaget, a pioneer of constructivism, developed the theory that it is detrimental to instruct a child in a given material before a given developmental stage has been reached. Unfortunately, as Stanislas Dehaene [18] describes, some of Piaget's experiments are badly flawed. For example, in a famous experiment of Piaget, young children were shown two rows of equally-spaced glass balls. When the spacing in the two rows was the same, the children stated correctly that both rows had the same number of objects. When the spacing was changed in one of the rows, the children said incorrectly that the longer row had more balls. On the basis of this experiment, Piaget and many educators have incorrectly claimed that even young children who can count have no number sense.

[3]In the United States and elsewhere this discussion has reached a wide audience. Articles in the *New York Times*, the *Wall Street Journal* and the *San Francisco Chronicle*, among others, report of widespread parental protests against the new math, MathLand, Connected Mathematics, and other projects inspired by constructivism (Hartocollis [34]; Saunders [69]; "Math wars" [52]).

Dehaene describes evidence refuting this assertion. When Piaget's experiment was repeated with chocolates –which of course the children could eat– instead of glass balls, they were found able to answer the second question correctly. Moreover, when a man dressed as a teddy bear rearranged the two rows while the interviewer's back was turned the children's performance on the second question improved.

The above illustrates the danger of making inferences concerning most effective educational practices on the basis of very small scale psychological experiments. The reality –ignored by many education experts– is that Project Follow Through, the largest and most extensive longitudinal study ever funded by the United States government in more than 180 U.S. schools (kindergarten through third grade) representing 51 school districts, demonstrated that the traditional Direct Instruction method was the most effective when compared to the other models tested, some of which were based on constructivist views. Students in the Direct Instruction groups performed better on both academic and affective (self-concept) measures. The Project, which cost almost a billion dollars and ran from 1968 to 1976 (Follow Through continued until 1995 as a federal service program), collected yearly data on around 10,000 students. Later evaluation of 1,000 graduates of the Direct Instruction groups showed that, in their senior year of high school, their performance was still better than their counterparts'. For details on Project Follow Through, see Kozloff and Bessellieu [47], Grossen [33], Watkins [82], Hirsch [35], and Carnine [10].

In Puerto Rico, constructivist, child centered, "progressive" educational policies have been in fashion for many years. From 1996 to 1999 the PR-DOE and the Puerto Rico Resource Center for Science and Engineering sponsored a regular column in the *San Juan Star* devoted to educational matters called *StarQuest*. The *Tips for Teachers* section of this column is quite revealing of the type of activity prevalent in constructivist math classes. On December 8, 1998, *StarQuest*'s central theme was religion. The age group to which this column is directed is not mentioned. The piece starts by giving a brief, what can only be called a caricature, of each of the world's major religions, and a description of the main festivals of each. We learn, for example, that the Hindus *"developed a group of gods"*. We also find the following advice for **mathematics (!)** teachers:

Teacher's Tips– Mathematics

> *Are religions basically similar to each other or different? Have your students analyze three major religions and make a list of what these religions have in common and in what ways they are different. Take the similarities and differences and convert them to percentages. What is the percentage of similarities to the total? Of the differences?* [sic] *Do the results agree with your students' original thoughts on this subject?*

One interpretation of the above piece of mathematical talibanism is that it is intended to teach percentages in the context of religion. However, there exist less innocuous construals. Whether intentionally or not, this article trivializes religious belief to such an extent that it could well have been written to undermine it. This suspicion is strengthened by the corresponding advice to science teachers:

Teacher's Tips– Science

> *To Buddhists, some monks can hear sounds far away, look into people's thoughts and levitate in the air... To Jews and Christians, God parted the waters of the Red Sea... To Hindus, Krishna is a god who lived on earth... Can these and other religious beliefs coexist with scientific thoughts and principles? How or why not?*

To persons of sound religious and scientific training, this question is highly non trivial. At the very highest levels of science, one can find both atheists and deeply religious people[4]. However, the typical student lacking this sophistication may predictably answer the question negatively. It seems that whereas the teaching **of** religion in schools is considered a violation of the separation of church and state, the teaching **about** religion in order to undermine it is considered acceptable.

This strain of teaching in "progressive" education has a long history. According to John Dewey [19] (quoted in Eakman [22]), one of the founders of this movement,

the great task of the school is to counteract and transform those domestic and neighborhood tendencies ... the influence of home and Church.

In the same article, he praises the "progressive" educational ideas of the Bolshevist government. Following the tradition of Dewey and others, many educators and psychologists have routinely used techniques such as cognitive dissonance and values clarification to change and mold students' belief systems (see Eakman [21], [22], Sykes [77], Gross [32], and Iserbyt [37]). The result is often ethical relativism as is well illustrated in the March 19, 1996 column of *StarQuest*, in which we find the statements:

Everyone's ideas are equally worthwhile... Suspend judgement: nothing is "right" or "wrong".

Neither is this a new paradigm in "progressive" education. According to John Dewey [20]

I cannot understand how any realization of the democratic ideal as a vital moral and spiritual ideal in human affairs is possible without the surrender of the conception of the basic division [good and evil] to which supernatural Christianity is committed.

As is well known, this type of illogic impales itself on its own contradictions. If it were really the case that everyone's ideas are equally worthwhile, there would be no reason to accept constructivism or indeed any other educational philosophy.

The tip for **mathematics** teachers in the March 19, 1996 edition of *StarQuest* is also typical of the sociological strain in modern constructivist mathematics education, so evident in the **PR-DOE Standards**. It reads

Teacher's Tips– Mathematics

The brain houses and integrates dissimilar functions in much the same way that the house has rooms for different daily activities. Draw a house with a room plan and yard. Draw lines (preferably of different colors) to connect rooms and areas with integrated functions [sic]. *(For example: Food prepared in the kitchen is eaten in the dining room. After sweating in the yard you take a bath and get dressed in your room.* [sic]*) Imagine one of the rooms being destroyed. What would be the impact on the family? Think of another model you can make that shows how things interact with each other.*

The above ludicrous example, far from being an aberration, is prototypical of the constructivist mathematical style widely referred to as Mickey Mouse Mathematics, Placebo Mathematics, New New Mathematics, and MTV Mathematics; and against which parents in California and elsewhere rebelled. The above examples are far from the worst. This mathematically vacuous column, if its advice was taken seriously, must have caused incredible damage to young minds. This may well be the case since exams at the University of Puerto Rico show that more than a few college students cannot identify the largest among 1/5, 1/6, 1/7 or, for that matter, multiply by 100.

[4] A case in point is Frank Tipler, a cosmologist and an expert in global general relativity. Tipler, who began his career as a convinced atheist, has recently written a five hundred page treatise [80] which attempts to prove that Judeo Christian beliefs are compatible with the laws of mathematics and modern physics. A serious account of the connections between religion and science is given by theologian and physicist E. L. Mascall in his book *Christian Theology and Natural Science*. An extensive bibliography on this subject can be found at http://www.newgenevacenter.org/sci-theo/bibliography2.htm.

3.7 Summary

Our analysis has shown that the **PR-DOE Standards** document is disorganized and badly formatted. In spite of being excessively long, it is not self contained. On the one hand, important topics are missing; on the other, some topics are endlessly repeated. It is not well sequenced. Many of the suggested activities are sociological rather than mathematical in nature. In general, the document expects too little from the student; well below international norms. In addition, it contains serious conceptual and mathematical errors. Specifically, regarding the benchmark attributes for standards given earlier, the **PR-DOE Standards** are:

1. **Not focused**
   Many of the standards (e.g., *Acquire the concept of fraction and its multiple representations* (page 30)) are too broad.

2. **Not specific**
   Some standards (e.g., *representar, analizar y resolver situaciones y estructuras matemáticas utilizando símbolos algebraicos*) are too vague. Different teachers are likely to interpret them in different ways.

3. **Not basic**
   The **PR-DOE Standards** are only partially successful in clearly identifying a subset of basic mathematics to be mastered by all students. Many important topics are missing. Basic principles (e.g., *equals added to equals are equal*) are omitted and instead there is too much emphasis upon student centered discovery learning.

4. **Not easily teachable**
   Besides the vagueness of some of the standards, the fact that the document as a whole is not logically organized, not well sequenced, and, especially, not self contained, detracts from its utility as a teaching guide.

5. **Not measurable**
   The **PR-DOE Standards** de-emphasize objective testing. Instead, there is a bias in favor of alternative assessment methods such as: comic strips, creative drawings, rubrics, poems, portfolios, and reflexive diaries. As already pointed out, besides being inefficient, these methods are very subjective and, hence, inadequate for summative evaluations of student performance.

6. **Not linked to grade**
   The **PR-DOE Standards** are not specific about what should be taught in each grade. They just merely list learning content for grade level blocks K-3, 4-6, 7-9, and 10-12.

7. **Not concise**
   The **PR-DOE Standards** document is over 250 pages long. Even at this length it is not self contained. Apart from grave organizational problems and unnecessary appendices, the lack of conciseness is due to the fact that certain words and phrases -usually vacuous- are constantly repeated like mantras (e.g., "vida diaria", "investigar", "construir", "conecta nuevas ideas matemáticas utilizando objetos concretos", "patrón", "modelar situaciones matemáticas con objetos", etc.). In addition, much time is wasted discussing the alleged merits of unproven educational theories.

8. **Redundant**
   The same topics repeat from standard to standard and from year to year with little or no increase in level (e.g., bar charts from grades K through 9!).

9. **Not genuine mathematics**
   The standards are not strictly mathematical but in many places are sociological in nature. There is an overemphasis on inductive (scientific) at the expense of deductive reasoning, which is the **essential** feature of mathematics.

10. **Not pedagogically neutral**
    The **PR-DOE Standards** are not pedagogically neutral but heavily loaded in favor of a constructivist educational paradigm.

4 Alternative Educational Models

4.1 PRSSI

At least four school "reform" projects are currently in progress in the public schools of Puerto Rico. These are:

- Expeditionary Learning/Outward Bound (ELOB)
- Computer Curriculum Consultants (CCC)
- Lightspan
- Puerto Rico Statewide Systemic Initiative for Excellence in Science and Mathematics (PR-SSI)

ELOB and CCC are relatively small projects. Lightspan –which involves the use of software run on a PlayStation (R) game console– has been adopted by 52 schools. PR-SSI seems to be the most comprehensive and visible of these initiatives and, according to PR-SSI data as of December 2000, has been implemented in 25% of public schools. Accordingly, the discussion that follows is confined to the latter.

According to a report of the State Higher Education Executive Officers Eisenhower Coordinators Network [78]

The SSI Program of the National Science Foundation was established in 1990 to encourage improvements in science, mathematics, engineering, and technology education through comprehensive systemic changes in the education systems. To receive five-year awards of $10 million, the Commonwealth of Puerto Rico committed matching resources (including Eisenhower Professional Development Program dollars) and **pledged** *changes in science and mathematics education (K-16).* [emphasis added]

These changes involve:

1. *The use of NCTM based standards and curricular materials.*
2. *The use of a constructivist approach involving cooperative learning, real world problem solving, etc.*
3. *Assessment of student learning.*
4. *Adoption of "new" methods and standards for the preparation and continuing development of teachers and administrators.*

The PR-SSI model was originally oriented toward the intermediate level (grades 7-9). In 1998, the reform was extended to elementary and high school levels. One positive result of this activity has been that high school graduation requirements in science and mathematics have been increased from 2 to 3 years, requiring students to take science and mathematics each year during high school. A somewhat less welcome outcome has been the adoption of the constructivist paradigm both by the PR-SSI and the PR-DOE. According to Manuel Gómez [31], director of the PR-SSI,

Students construct their own knowledge; it cannot be given to them.[5]

In 2000, a change of government occurred and it is not yet clear whether PR-SSI will be extended to the entire public school system. That it possibly may not -at least in its present format-, is evidenced by the fact that in the *Catálogo de Cursos para Maestros de Nivel Superior* [26], the three compulsory years of Mathematics, Science, English and Spanish have been reduced to only two. This is difficult to comprehend, especially given the fact that one of the primary authors of the new catalog (and former Subsecretary of the Department de Education), Ana Helvia Quintero, is a Co-Principal Investigator in the PR-SSI [64]. It is abundantly clear from the new course catalogue that the same constructivist paradigm will be emphasized by the new administration. Besides, in view of the millions of federal dollars given to Puerto Rico in exchange for promises to implement the constructivist "reforms" mentioned above, it is not clear how much freedom of maneuver the PR-DOE actually has to change current educational policy. According to Luther S. Williams [58], a previous National Science Foundation (NSF) Director of Education and Human Resources (under whose term of office the PR-SSI program was initially funded), *"We need to change the attitude that math and science are optional."* It may well happen that reneging on commitments to the NSF and making math and science optional will have pecuniary consequences for Puerto Rico. After all, this is the very same Dr. Williams who once threatened to cut off NSF funding to California for adopting a non NCTM set of standards [70].

During a meeting of the Faculty of Natural Sciences of the Río Piedras Campus on April 12, 2002, the former Subsecretary of Education announced that PR-SSI materials would be integrated into the new curriculum. If this is to be the case, their effectiveness should be assessed. According to an evaluation of the NSF's Statewide Systemic Initiatives Program [48]

*In most states it is impossible to trace a pattern of improved achievement that is sustained across more than one year of testing... Evidence of gains in student achievement is uneven or contradictory. In four of the seven states reviewed here, gains in student achievement were not sustained across grades or were not consistent from one year of testing to the next. In many cases, effect sizes are small. Using one-third of a standard deviation as a rough guide for an improvement in scores that approaches educational significance, it appears that only Montana and (***perhaps***) Puerto Rico met this standard. However, because the SSIs did not assess effect size and because they did not publish information about the variance of individual school or student scores, it is difficult to be sure. ... In light of the nature of the SSIs' "treatments" of students, the numbers of teachers involved, and the data examined in this report, it seems likely that the SSIs' impacts on student achievement were limited, both in the numbers of students affected and in the size of any gains.* [emphasis added]

Moreover, the **small** improvement claimed by the Puerto Rico SSI should be taken with a pinch of salt. Due to failure to control for significant factors, that conclusion is meaningless. Apart from an

[5]Incredible as it may seem, there are even more extreme constructivist viewpoints. In a paper in the *Journal for Research in Mathematics Education*, Cobb, Yackel, and Wood [13] proclaim

"it is possible for students to construct for themselves the mathematical practices that, historically, took several thousands of years to evolve."

One wonders how this inanity passed by the referees.

element of self selection by the schools themselves, many of the PR-SSI schools were simultaneously involved in other projects such as Lightspan, so it is difficult to correctly attribute any responsibility for measured improvements. A case in point is the University Gardens High School, one of the more successful schools of the PR-SSI project. This school was converted, amidst great opposition from local residents, to a specialist Science and Mathematics School with stronger admission criteria. In addition, it has a history of close contacts with the University of Puerto Rico's Mathematics Department via projects such as CRAIM (Regional Center for Mathematics Research and Training) in which university professors closely supervised student research projects in this school. Thus, it is all but impossible to substantiate the claim that PR-SSI alone –or at all– is responsible for an improvement in student outcomes.

In effect, there is little evidence that SSI programs have achieved one of their major goals, namely, improving student outcomes. However, as noted on page iv. of the same report

Changing state policy and aligning the components of state education systems were important goals, as well.

It appears that the main result of 10 years of sustained effort and 10 million dollars has not been the significant improvement in student outcomes but rather a realignment of the Puerto Rican public education system with the constructivist paradigm and the failed standards of the NCTM described in the next section. Constructivism has become an obsession in Puerto Rican educational circles.[6] Sadly, after all of the reform efforts of the last 10 years, the majority of students who are admitted to the Natural Sciences Faculty of the UPR Río Piedras Campus are at a mathematically very low level. This is evidenced by the abysmal results of the annual placement exam given by that Faculty.

4.2 NCTM Standards

Both the **PR-DOE Standards** and the PR-SSI initiative have been strongly influenced -for the worse- by the Standards of the National Council of Teachers of Mathematics (NCTM Standards [55], [56]). The NCTM Standards were controversial from the moment of their introduction and were far from universally accepted within the mathematics community. In reality no experimental studies had been done to validate the NCTM standards. The justification provided for their introduction was at best anecdotal.

According to Frank B. Allen [1], former president of the NCTM,

The standards-based subject purveyed by the NCTM is so laden with major defects, so over-adjusted to alleged student learning deficiencies, that it no longer retains the properties of mathematics that make its study worthwhile.

The NCTM Standards have been correlated to lower mathematics skills. In the United States, the worst impact has been on Hispanics and blacks. For an evaluation of the NCTM Standards, see Wu [85], [86], Carnine [10], Schommer [72], and articles in the Web site http://mathematicallycorrect.com/. A report of the Council of the London Mathematical Society (quoted by Wu [85]) on similar changes in the United Kingdom points out that,

we can look forward to a generation of students with:

[6] See, for example, the article by Canny Bellido entitled *Arriba el Constructivismo!* in the Bulletin of the Puerto Rico Collaborative for Excellence in Teacher Preparation [5], in which we even find a poem with the lines

El constructivismo
Es nuestro objetivo
Pertinencia e integración
La de nuestros niños.

Thus, constructivism has become not only a means to an end but also the end.

i a serious lack of essential technical facility–the ability to undertake numerical and algebraic calculation with fluency and accuracy;

ii a marked decline in analytic powers when faced with simple problems requiring more than one step;

iii a changed perception of what mathematics is–in particular of the essential place within it of precision and proof.

Math scores in California, one of the first states to adopt curricula based upon the original NCTM standards, plummeted to the third lowest in the nation. In the 1996 National Assessment for Educational Progress (NAEP) math test, California's fourth graders ranked fourth worst in the nation trailed only by Mississippi, Guam, and the District of Columbia.

The NCTM "standards" led to the so called "Math wars" [38] across the U.S. and to massive parental rebellion in California. In 1995 the California Board of Education decided to abandon the NCTM standards and produce their own. In 1997, despite threats from Luther S. Williams [58] (who is not a mathematician) to cut NSF funding, California replaced the NCTM Standards by the new document *Mathematics Content Standards for California Public Schools, Kindergarten through Twelfth Grade* [9]. For a detailed discussion of the background to this controversy see Wu [86].

5 International Comparisons–TIMSS

The Third International Mathematics and Science Study (TIMSS) [54] of 1999 is the largest international study of student achievement in mathematics and science ever conducted. It involved more than half a million students in 38 countries. Testing was done at five grade levels and studies were made of teaching methods, curricula, textbooks, technology, as well as sociological factors. Classroom activities were videotaped in the various participating countries. Since 26 of the 38 countries had also participated in TIMSS 1995 it was possible to measure trends and changes in relative performance. Although Puerto Rico did not formally participate in the TIMSS survey, many of the findings of the TIMSS are relevant for Puerto Rico. In particular, since curricula and textbooks in Puerto Rico are heavily influenced by their U.S. counterparts, the performance of the U.S. in this survey is of great concern.

According to the TIMSS report [54], U.S. seniors ranked 19th out of 21 nations in general mathematics. Only Cyprus and South Africa fared worse. Moreover, many top performing nations such as Japan, China, and Singapore (ranked top at eighth grade) did not participate in the senior study.

According to William Schmidt [71], executive director of TIMSS,

The most important message we can derive from the TIMSS results is that "curriculum matters"... We need to address what gets covered, in what sequence topics are covered, and how subject areas are integrated throughout a student's education... Our current curriculum lacks coherence and rigor. First, our textbooks and courses lack any focus. 8th grade science textbooks, for example, attempt to cover some 65 topics over the course of a school year. The international average for the same grade level is 20 topics. ... Our current curriculum is also highly repetitive... Grades 5 - 8 are a wasteland from the point of view of math and science because topics such as simple arithmetic are simply rehashed grade to grade; students no longer face any intellectual challenge. These grades also represent the period when TIMSS scores start to slide downward and should accordingly be the initial focus of changes in building a science framework... Two final ingredients must be included *in a new science framework: access to the new curriculum for all students and professional development for teachers. All teachers must have a firm background in their subject area.*

According to R.B. Schwartz [73],

The important lessons of TIMSS are to be found not in the comparative ranking of the countries but in the extraordinary sub-studies that accompany the administration of the tests. The first study focusing on textbooks strongly suggests that, in the absence of clear agreements about what students are supposed to know and be able to do at each grade or cluster of grades, our textbooks err on the side of inclusiveness, treating a large number of topics superficially rather than a handful of topics in depth. The second study examines videotaped classrooms in Germany, Japan, and the U.S., and this is enormously instructive in what it reveals about the focus on pedagogy in the three countries. Simply put, the American lessons, especially when contrasted with Japanese classrooms, focus much more on procedures and skills, and much less on concepts, deductive reasoning, and understanding.

Regarding the use of calculators, the TIMSS report [54] indicates that in the fourth grade, in which the U.S. did comparatively well, in six of the seven nations that outscore the U.S. in mathematics, 85 percent of the students **never** use calculators in class.

Commenting on the TIMSS results, the *Economist* [83] reports that Julia Whitburn of the UK National Institute of Economic and Social Research has studied mathematics teaching in Japan and Switzerland, both high performing countries in mathematics education. She has noted a number of factors common to both countries:

- *Much more time is spent on the basics of arithmetic than on more general mathematical topics such as handling data.*
- *Pupils learn to do sums in their heads before they are taught to do them on paper. Calculators are usually banned.*
- *Teachers use standardized teaching manuals, which are tested extensively in schools before being published.*
- *Whole-class interactive teaching is used widely; i.e., the teacher addresses the whole class at once, posing questions to pupils in turn, to ensure they are following the lesson.*
- *Great efforts are made to ensure that pupils do not fall behind. Those that do are given extra coaching.*

6 Post Data- The 2002 Curriculum Reform

The PR-DOE is currently proceeding with a reform of the high school curriculum. The documents serving as basis for this reform are: Carta Circular Núm 30-2000-2001, Carta Circular Núm 9-2001-2002, amended in Carta Circular Núm 11-2001-2002, as well as the *Catalogue of Courses for High School Teachers* [26] (the **Catalog**).

6.1 Some Aspects of the Reform

A significant aspect of the reform is that the three compulsory years of Mathematics, Science, English, and Spanish have been reduced to only two and the course sequences within the first two years of mathematics and science made electives. In effect, there will be **no** well defined mathematics core. In the third year, instead of mathematics, the student will be free to choose between such condescendingly named courses as: *Trópico-Música (Ayer y Hoy)*, *Quím_era.com*, and *Musas/Dinámicas.com.*

Let us consider the case of a hypothetical student who chooses the following math sequence:

	Mathematics Course	**Prerequisites**
Yr. 10 - Sem. I	*La Estadística y Probabilidad en la Vida Diaria*	
Yr. 10 - Sem. II	*La Estadística en el Campo de la Investigación y Nociones Probabilísticas*	*La Estadística y Probabilidad en la Vida Diaria*
Yr. 11 - Sem. I	*Finanzas del Consumidor*	
Yr. 11 - Sem. II	*Lo plano y algo más*	

Suppose that in the twelfth grade, our hypothetical student completes his course work with *Sex@**#@.com*, *Baila con Estilo*, *Tecno-Música Contemporánea*, and *Taekwondo*. His job and further study options are at this point quite limited. With the math knowledge obtained from the courses listed above, he would be unprepared even for the Puerto Rico College Board test in mathematics -a very low level exam- which is required for college admission. Even for vocational work such as construction or carpentry, his knowledge of geometry –needed for reading architectural plans, etc.- would hinder his progress.[7] A true educational reform should guarantee that all students are **challenged** to achieve the **maximum** they are capable of.

The proposed "reform" also effectively abolishes the usual classification of the disciplines first begun by Aristotle. For example, the **Catalog** includes the following aliases:

Álgebra:	Expresiones más allá de los números
Geometría:	Lo plano y algo más
Literatura:	El hombre: Sus inquietudes y aspiraciones en el espacio vital
Física:	Si te mueves, te energizas
Biología:	Desde la creación al hombre
Química:	Quím_era.com
Música:	Musas/Dinámicas.com
Teatro:	Dramatízalo conmigo
Historia:	Estados Unidos a través del tiempo
Salud:	Sex@**##.com

The corresponding dissolution of the disciplines reflects an obsession for the integration of everything with everything else, including the so called "real world", which actually leads to **disintegration**. The concomitant cutting of course content is well illustrated by the description of the course *Comunicándonos Matemáticamente:*

> *El estudiante aprende y amplía su lenguaje matemático. Se establecen conexiones con otros conceptos matemáticos, con otras materias y con el mundo real. Se inicia con la exploración de ideas hasta llegar al formalismo matemático. Su área de énfasis será* **relaciones**, *específicamente el álgebra enmarcada en un enfoque de solución de problemas.* [sic]

This most peculiar description leaves one with the distinct impression that it was written by a person of rather low mathematical level. It is so vague that students, guided by their teachers ("facilitators"), will be forced to construct not only their own knowledge, but also their own course content.

[7]For the sake of comparison, a student in the vocational program in Holland receives 360 hours of mathematics instruction despite having only four years of secondary education.

Although the focus of this article is mathematics, I cannot help pointing out another strange feature of the proposed curriculum. Two of the three proposed "Physical" Education courses (the third is Taekwondo) are "theoretical" in nature, requiring no actual "physical" activity on the part of the student. One is called "Organization of Sporting Activities and Events"; the other, "Refereeing". In the latter, the student is expected to study the rules of the sports practised in the Physical Education Program. It is wishful thinking to believe that "studying" the rules of sports will equip a student to be a referee or even an interested spectator. The most effective way to understand a particular sport is to **physically** take part. How paradoxical that "hands-on application" -a constructivist theme- is absent from this one subject where it is almost necessary.

6.2 Basis for the Reform

Essentially three types of justification have been proffered for this reform.

(c1) Philosophical justifications.
(c2) Concern with attrition rates.
(c3) Reforms in Japan and Holland.

In this section we provide a brief examination of each.

6.2.1 Philosophical Justifications

According to the **Catalog**, page 1,

Se están desarrollando modelos de integración curricular y aprendizaje constructivista, de manera que los currículos sean pertinentes, basados en las necesidades de los alumnos, innovadores y creativos.

As far as mathematics is concerned, this is not really a change. Both, the NCTM inspired **PR-DOE Standards** and the **Catalogue** are based on the constructivist paradigm. Besides, as previously pointed out, the PR-SSI "reform", which has operated in many schools for a number of years now, is also constructivist. Similar remarks apply to other disciplines. Constructivist, child centered education has long been the dominant paradigm in almost all Schools of Education in both the U.S. and Puerto Rico.

According to the former Subsecretary of Education, Ana Helvia Quintero ([66], pages 1-2),

El desarrollo de un currículo debe partir de las investigaciones realizadas sobre los procesos de aprendizaje. En las últimas tres décadas ha habido gran actividad en estas investigaciones. De éstas surgen cuatro principios básicos sobre el aprendizaje:

(p1) *El ser humano, más que buscar información, busca aprender con significado. (Bruner [6]).*

(p2) *El aprendizaje con significado no sigue únicamente el proceso analítico-racional. (Lakoff [49]).*

(p3) *Existen "múltiples inteligencias" y las personas aprenden de formas diversas de acuerdo a su fortaleza. (Gardner [29]).*

(p4) *El desarrollo socio-emocional tiene una estrecha relación con el desarrollo cognoscitivo. (Damasio [16]).*

From these statements is inferred a need for, among other things, "learning with sense", "in context", "interdisciplinary learning", and "the organization of the curriculum around themes ", etc., -hardly new ideas. Yet, the above claims and conjectures are so broad that it is far from clear how they relate to classroom practice and curriculum content. For example, as has already been discussed in section 3.6, Bruner's thesis [6], that "meaningful" learning can **only** occur in some "context", is not in accord with the evidence from cognitive psychology (Anderson et al. [2]).

In his book, *Women, Fire, and Dangerous Things* [49], Lakoff conjectures that human cognition is vitally dependent on *metaphor*, which he defines as a mapping of conceptual structures from one domain onto another. In their recent book [50] Lakoff and Núñez extend this idea to the realm of Mathematics. They adopt the position that mathematics also arises from cognitive processes in which the concept of *metaphor* plays an important role. An example of such a metaphor is the well known "map" between sets of objects of the same size and the abstract notion of number. In their attempt to find metaphors in more advanced settings, the authors make serious mathematical errors (see the review by Auslander [4]) which undermine their thesis. Moreover, their position directly challenges the Platonist viewpoint held by many mathematicians that mathematical ideas are discovered.

Gardner's theory of multiple intelligences is a reasonably sounding idea. It is obvious that people have different strengths and are better at some things than others. However, it is a big leap from this raw idea to the concept of specific, independent intelligences. Gardner's theory is not founded on experiment. According to James Collins [14]

> *evidence for the specifics of Gardner theory is weak, and there is no firm research showing that its educational applications have been effective.*

Collins goes on to point out that

> *the most common use of Multiple Intelligence is to attack a topic from seven directions to fit in all the intelligences. Take a typical project described in a book published by Sky-Light. To teach children about the oceans, it is suggested that they write about cleaning a fish (tapping the linguistic intelligence), draw a sea creature (spatial), "role play" a sea creature (bodily-kinesthetic), use a Venn diagram to compare and contrast ships (logical), tap glasses with different amounts of water (musical), design a water vehicle in a group (interpersonal) and choose a favorite sea creature (intrapersonal). All these activities will take up a lot of time, and they will teach children very little about the ocean.*

Gardner himself has admitted that there is no firm research showing that his theory's practical applications have been effective (See Collins [14] and Gardner [30]).

On the other hand, Damasio [16] cites modern studies of brain damaged patients to demonstrate that emotion and rationality cannot exist separately. Once again, this extension of Descartes' "Cogito ergo sum" is too far removed from curricular matters to justify the educational reform proposed by the PR-DOE. Much evidence is needed before translating experiments on brain damaged patients into curricular practice.

Clearly, a better understanding of the cognitive sources of mathematical ideas can help us learn and teach. However, the precise implications for curricular reform await long term longitudinal studies. We should not with almost indecent haste replace traditional methods which have worked well over time with psychological or philosophical "theories" of unproven value.

6.2.2 Concern with Attrition Rates

In addition to the above philosophical justifications for the reforms, the former Subsecretary of Education has claimed that the current courses are considered "boring" and, as a result, students are dropping out of school. Since the new courses are based on the same constructivist philosophy and, in some cases, even have similar content to existing **PR-DOE** courses, it is not clear how the new courses will be less "boring" than the old. Doing laundry can also be "boring" but no one suggests making laundry optional. In the case of mathematics, our analysis clearly shows that the current courses may well be boring and probably often badly taught. However, one can scarcely say from the scant, vaguely described courses in the **Catalog** that the new courses will be any better. In any case, the answer is not to replace a “boring” math course with a course in the supposedly less “boring” *Baila con Estilo* but to **improve** the mathematics course.

There are many reasons why students drop out. These include: socioeconomic factors, frequent teacher absence, disciplinary problems, and lack of adequate counseling. It could even be that some leave school because, as our analysis of the **PR-DOE Standards** has shown, courses are not challenging enough. One of the best predictors of performance in later grades and, hence, of low attrition, is first grade reading ability (see section 7). However, the so called whole language method for teaching reading, prevalent in many schools, is known to be inferior to phonics instruction and for many students does not even work for neurological reasons. Such students are at a much higher risk of dropping out or falling into delinquency and crime. Finally, most students drop out in **intermediate** school so one does not see how modifying the **high** school curriculum will help this group.

6.2.3 Reforms in Japan and Holland

Another reason cited for the reduction in math courses is a Japanese proposal[8] to reduce the amount of Mathematics studied in high school starting in 2002. The essential logic seems to be that if the Japanese commit an error then so should Puerto Rico. Actually, the error will have considerably greater consequences for Puerto Rico than Japan. In the 2000 International Mathematics Olympiads, Japan was fifteenth out of eighty two nations with a total score of 125/252 and Puerto Rico was eighty second -in fact last- with a score of 8/252. In reality, over 97% of all Japanese students successfully complete Mathematics 1, the Japanese grade 10 course [41] which covers much of the material covered in Puerto Rico in the **university** under the aegis of Precalculus. In Japanese grade 11, 40% of students take the easier General Mathematics 2 course [42] which covers “probability and statistics, vectors; exponential, logarithmic and trigonometric functions, as well as an informal presentation of calculus”. The remaining 60% of students take **two** more extensive courses, Algebra and Geometry [43] and Basic Analysis [44]. These courses cover “plane and solid geometry, vectors and matrices, additional trigonometry, as well as differential and integral calculus”. So, by grade 11, nearly **all** Japanese high school students have considerably more mathematics than is required of a Natural Sciences graduate in, say, Biology at the University of Puerto Rico. It is sophistry to suggest that because some Japanese wish to do less mathematics this would also be good for Puerto Rico.

Arguments have also been put forward that Puerto Rico should adopt elements of the Dutch Model of Mathematics Education. However, many of the factors which contribute towards the success of the Dutch Model are not present in Puerto Rico and cannot not be easily established. For instance, as many as 70% of schools in the Netherlands are privately owned, and often nominally religion-based. These schools are also publicly financed. The number of years of compulsory education is not fixed but varies from ten to twelve. Secondary education is divided into four options ([81]):

1. **4 years pre-vocational education**
   This option is usually terminal. The student specializes in technical education, home economics, a commercial trade or agricultural studies.

[8] This proposal has met with massive opposition in Japan. At the opening ceremony of the Ninth International Congress on Mathematical Education in 2001, Prof. Sugiyama, President of the Japan Society of Mathematics Education, criticized the upcoming reduction.

2. **4 years general secondary education**
   This four year program is often followed by an apprenticeship or vocational training.

3. **5 years general secondary education**
   This track leads to higher professional qualifications. It is possible for a student to transfer from this option to the pre-university track.

4. **6 year pre-university education**
   This option is designed to prepare students for university entrance. There are two types of school offering this program, the *atheneum* and the *gymnasium*. The athenea are public schools. The gymnasia are smaller independent schools with similar curricula except that they also include Latin and Greek.

The option to be taken is chosen in the final year of primary education, at age 11, and is based upon a compulsory exam. A student in the 6 year pre university option takes 760 hours of mathematics including 320 hours of calculus and 160 hours of geometry. The rest of the time is taken up with functions and graphs, trigonometry, discrete calculus, combinatorics and probability, continuous dynamic models, and 40 hours on the normal distribution and inferential statistics. At the other extreme, the pre-vocational option includes 360 hours of mathematics. Students in this option leave at age 16 with a reasonable preparation. Unlike in Puerto Rico, percentages are taught at the primary level; since 1993 they are also taught in a more sophisticated manner at the secondary level. In order to teach at the pre-university or higher secondary levels, either a five year university degree or a four year degree followed by a year at a college of education is required.

It can be seen from this brief presentation that the Dutch system (which is itself in a state of change) is so different from the Puerto Rican model that attempts to adopt even parts would have to be done with great caution and experiment. Consequently, we warn strongly against any attempt to implement portions of this model on a large scale in Puerto Rico as is being considered.

6.3 Summary

The reform of the curriculum is being pursued with excessive haste, without careful planning, and with inadequate justification and experimental basis. This can only increase the existing curricular chaos. In the case of mathematics, the reduction of credits, the lack of a well defined core, the lack of a logical course sequence, and the mediocrity of the course descriptions suggest, among other things, that experts in Mathematics were not consulted before producing the new **Catalog**. It is disturbing that the ample literature concerning curricular reform has not been adequately examined. If the curricular reform is implemented as planned, it is very likely that the already large gap in performance between public and private school students in Puerto Rico will be further widened.

7 Reading Instruction in Puerto Rico

Success in mathematics instruction and, indeed, in all disciplines is critically dependent on the child's reading ability (Chall [11]). According to Davis et al. [17] arithmetic performance is highly correlated with a combination of phonemic awareness (the ability to distinguish phonemes) and memory. Accordingly, any reform in the mathematics curriculum should be coupled with an examination of reading instruction.

The current method of reading instruction in Puerto Rico is based on the *whole language* approach [27]. This approach assumes that written, like spoken language, is a human "instinct" which can be taught by exposing the student to an appropriate reading environment in which actual texts are read to and by the child. Unfortunately, scientific evidence shows clearly that this is not best practice and actually fails to varying degrees in 20% to 40% of the population. Well replicated medical research using neuroimaging and other technologies has found a unique brain signature in many of these persons (Pugh et al. [65]). Phonics instruction, on the other hand, has been shown to be effective in most such cases.

That the creation of written language is not natural is also evidenced by the fact that, historically, many cultures never developed one. Many languages remained unwritten until relatively recent times and, even then, the written form was often introduced by non native speakers. According to Steven Pinker [63], professor of Brain and Cognitive Sciences and director of the Cognitive Neuroscience Center at MIT,

Until recently, most children never learned to read or write; even with today's universal education, many children struggle and fail. A group of children is no more likely to invent an alphabet than it is to invent the internal combustion engine. Children are wired for sound, but print is an optional accessory that must be painstakingly bolted on. This basic fact about human nature should be the starting point for any discussion of how to teach our children to read and write.

In 1997, the U.S. Congress asked the National Institute of Child Health and Human Development to examine the scientific evidence concerning how children learn to read. As a result, the National Reading Panel was created. This Panel considered the findings of over 100,000 experimental studies published since 1966 and more than 10,000 before. Only those studies meeting standard criteria for scientific rigor (the use of controls, replicability, etc.) were used in formulating conclusions. According to the Panel's Report [57]

- *teaching phonemic awareness to children significantly improves their reading;*
- *understanding the relations between sounds and letters helps students from K-6th grade and children having difficulty learning to read;*
- *vocabulary development, background knowledge, text comprehension strategies are all critical to the development of reading comprehension; and*
- *guided oral reading procedures that include guidance from teachers, peers, or parents have a significant and positive impact on word recognition, fluency, and comprehension.*

There is nothing in this Report to support the premise of whole language proponents that reading, like spoken language, is instinctual. Most of the experiments which proclaimed the superiority of whole language are anecdotal in nature and not replicable. In fact, when whole language was widely adopted in California in 1987, National Assessment of Educational Progress (NAEP) scores fell dramatically. By 1994, California NAEP scores were among the worst in the U.S. Recognizing that the introduction of whole language instruction had been a mistake, on September 12, 1995, the California Legislature unanimously passed bill AB170, mandating that materials for reading instruction include **systematic, explicit phonics.**

Thus, the main principles and practices specific to a whole language approach to reading are not corroborated by **rigorous** scientific studies; those of systematic phonics are. Why, therefore, is Puerto Rico emphasizing whole language years after it has been abandoned elsewhere? Besides California, many other state governments as well as the federal government have already recognized the necessity of phonics instruction. For example, in July 1998, Wisconsin enacted a law which requires that **any** applicant for a license to teach reading or language arts in K-5 must have successfully completed instruction that prepares him or her to teach reading and language arts using appropriate instructional methods. These **must** include phonics. Puerto Rico cannot have successful mathematics instruction for all students while some are held back by reading problems. Worse, low reading literacy has been associated with desertion from school and is an important predictor of criminal behavior. Studies concerning incarcerated juveniles, by Michael Brunner [7] of the U.S. Department of Justice, concluded that **failure to read** is by far the strongest statistical predictor of violent behavior – much stronger than poverty, drugs, broken homes, or other sociological factors. Brunner argues that this correlation is causal. Thus, reading problems are more likely to be the cause of the high attrition rates in Puerto Rican schools than allegedly "boring" mathematics classes.

8 Recommendations

A series of recommendations can be inferred from the previous analysis. The following is a summary of the main ones organized according to: standards, educational philosophy, teachers and teacher training, textbooks, and, finally, technology.

8.1 Recommendations Concerning Mathematics Standards

We urge the PR-DOE to appoint a permanent Mathematics Standards Panel to produce and maintain PR-DOE Mathematics Content Standards that replace the current **PR-DOE Standards**. The Panel should include qualified **practising** research mathematicians. It should explicitly exclude persons with actual or potential conflicts of interest (e.g., authors of textbooks which are under consideration, recipients of educational awards and grants for developing and disseminating particular educational philosophies or teaching models which, if implemented, would result in pecuniary gain).

The new Content Standards should be focused, specific, basic, teachable, measurable, linked to grade, concise, pedagogically neutral, mathematically correct, internationally competitive, and well written. A 1998 Fordham Foundation [68] review of mathematics standards ranked the new California standards top among 46 states and better even than those in Japan. We propose that these standards (or a significant subset thereof) be examined as a possible model on which to base the new PR-DOE Mathematics Content Standards.

The Mathematics Standards Panel should also be in charge of producing a Puerto Rico Mathematics Framework aligned with the content standards. Objectives of the Framework should include: provision of **research-based** information about how children learn and corresponding instructional strategies; guidance for developers and authors of instructional resources to ensure alignment with the Mathematics Content Standards, and specification of a reasonable balance between basic computational and procedural skills, problem solving, and conceptual understanding. The Framework should also specify the minimum core knowledge for entering university, pursuing a scientific or mathematical career, taking advanced placement tests, etc.

We recommend that the PR-DOE Mathematics Content Standards and Framework be covered at length in the Schools of Education of all P.R. universities -public and private. The Panel would contribute by also identifying effective, **research-based** mathematics **teaching** standards. Kozloff [46] provides the following examples of such standards:

1. *The teacher can list the types of errors students might make in multiplying three-digit numbers.*

2. *The teacher can describe error correction formats for each type of error in multiplying three-digit numbers.*

3. *The teacher can provide a sequence of examples of three-digit multiplication problems showing a logical progression.*

The Panel should devise mechanisms for assessing the progress of Schools of Education in implementing the proposed model in their curricula, which should be part of their accreditation criteria. Finally, the Panel should assess the various public exams, such as the Prueba Puertorriqueña and the College Board, and ensure alignment with the Standards.

We recommend that the PR-DOE Mathematics Content Standards and Framework be made widely available through libraries, bookstores and the internet so that teachers, students, and parents are aware of what is expected at each grade level. To this end, a **politically neutral** web archive for official PR-DOE Curricula and Standards, etc., should be set up. This should be independent of the Department of Education's official web site –which essentially disappears after each election to be later revamped according to the current political ideology. Content of this new archive should be limited to official curricula and standards.

8.2 Recommendations Concerning Educational Philosophy

We recommend that the PR-DOE exercise greater caution in adopting unproven mathematics programs merely because they are labelled *best practice* and/or are heavily funded by the U.S. Department of Education or other federal or private entities. Unfortunately, the term *best practice* is widely used by educators to mean teaching practice that is consistent with the constructivist educational theories currently in vogue, not that which is actually the most effective. As discussed in section 3.6, major longitudinal studies such as Project Follow Through (Watkins [82], Carnine [10], Grossen [33]) and the work of cognitive psychologists Anderson, Reder, and Simon [2], among many others (see Chall [11], Hirsch [35]), demonstrate that constructivist based educational methods are **not** educational "best practice". By the same token we urge the PR-DOE to de-emphasize the influence of the inadequate mathematics standards of the NCTM, which are not experimentally based. Instead it should promote the use of whole class instruction methods such as Direct Instruction which have a sound experimental basis (Chall [11], Hirsch [35]).

Likewise, we recommend that the Prueba Puertorriqueña not be aligned with the National Assessment of Educational Progress Test (NAEP). The latter is considered by many (see Eakman [21], [22], and Iserbyt [37]) to be a psychological test masquerading as an academic test. Moreover, as a result of its adherence to NCTM philosophy, its level, particularly in mathematics, is very low and it omits many basic and important topics. According to detailed testimony of economist John Hoven [36] to the National Public Forum on the Draft 2004 Mathematics Framework for the NAEP, "hard 8th grade problems, -according to NAEP's classification- are at the level of Singapore's grade 5 or lower".

We urge the PR-DOE to introduce intensive phonics-based reading instruction and, to the extent possible, prior to grade 1. Students cannot do math if they cannot read. Given the fact that "phonics instruction facilitates early reading acquisition is one of the most well established conclusions in all of behavioral science" (Stanovich [76]), Schools of Education that fail to teach and emphasize phonics-based reading instruction do not deserve to receive government funds. Students failing to read by grade 1 should be given intensive extra instruction in phonics-based reading.

8.3 Recommendations Concerning Teachers and Teacher Training

Schools of Education in Puerto Rico attract few applicants in the field of mathematics, and many of those have poor mathematics skills. According to statistics presented in *El Nuevo Día* [15], in the 11 years from 1991 to 2001, only five persons graduated from the University of Puerto Rico with concentrations in Elementary Mathematics. In 1998, the number of graduates in Mathematics Education (both elementary and secondary) was only 39 for the whole of Puerto Rico. This is partly due to low pay, poor working conditions, and to the low status our society sadly grants the teaching profession. The shortage of mathematically qualified teachers exacerbates the current catastrophic situation in the schools. Accordingly, we recommend that steps be taken to increase teachers' pay, particularly in difficult to recruit areas such as mathematics. The best teachers, as determined by performance in a competitive exam in the subject matter and by evaluation of their effectiveness in the classroom, should be eligible for merit pay and other incentives. In order to attract more mathematics majors into teaching, scholarships could be awarded which are **linked** to a **legal** commitment to teach for a certain number of years.

We recommend that persons with a degree in mathematics and good grades be allowed to teach in a probationary capacity while they take the necessary education courses. Extra supervision should be provided until the candidate teacher has passed the necessary courses and demonstrated teaching competence. Variants of this idea have been tried in New York with some success. In many countries, such as the United Kingdom, a graduate in any discipline can take a full time one year program leading to full teacher certification as an alternative to the usual four year education degree. This scheme has worked well and attracted many graduates into teaching.

The issue of mathematical competence must also be addressed. Unfortunately, having "good" grades from public or private universities in Puerto Rico, as elsewhere, is not a guarantee of competence in mathematics. Too many university professors give students grades which they do not deserve or, even worse, give grades merely for attendance. As a result, students enjoy "good" grades, "good" transcripts, and "good" diplomas, while professors gain and enjoy the reputation of being "good" educators and proclaim as much in their résumés. As is well known, a good looking résumé leads to fringe benefits such as promotions, prizes for excellence, sabbaticals etc. Unfortunately, many teachers of mathematics lack even "good" grades.[9] Accordingly, we recommend that the PR-DOE require a rigorous qualification examination for all new mathematics teachers including math majors. No one should be allowed to teach mathematics who cannot demonstrate minimal competence in this exam.

In the case of education majors, we recommend more extensive course work in the student's **academic** discipline and a corresponding decrease in the number of those courses not directly linked to the discipline itself. One of the current problems is that many teachers possess all the formal requirements to teach mathematics yet their mathematical level is too low. Sadly, psychology and methodology courses will not compensate for a lack of knowledge in the academic discipline.

In Puerto Rican universities, both public and private, admission requirements for Education programs are among the lowest of all disciplines. Consequently, students in Secondary Mathematics Education often have serious gaps in their high school mathematical background and are, in general, totally unprepared for the traditional mathematics sequence Calculus I, Calculus II, Abstract Algebra I and Linear Algebra. This is particularly true for the huge conceptual leap between Calculus II and the two Algebra courses. Extremely high failure rates in Abstract and Linear Algebra attest to this. Students who do pass often have low grades and poor understanding of the material. We recommend that this conceptual gap be filled by the introduction of two additional courses, one in the Foundations of Mathematics and a continuation course in Discrete Mathematics and Combinatorics, to be taken as prerequisites to Abstract and Linear Algebra. Such courses are now standard first year courses in many world-class universities and, indeed, much of this material is

[9] A mathematics education student in my Linear Algebra class a few years ago had taken over thirty (including repetitions) math courses all with grades D, F or W. He did, however, have an A in the Teaching of Mathematics and is probably now teaching mathematics in a high school near you. I have even encountered graduate mathematics majors who do not know trivial topics from precalculus.

actually taught in the high schools of mathematically successful nations. It is very important that teachers have a more solid and coherent sequence of mathematics courses covering the topics they are likely to teach.

Teachers already in the system should be encouraged and, if necessary, required to take courses in their area of expertise and approve them with good grades. A grade of C or D in a precalculus or calculus course should not qualify a student to teach high school mathematics. Therefore, teachers with low grades in key courses should be encouraged to repeat the corresponding courses. On the other hand, we recommend that the number of teacher workshops, in which nothing but attendance is required, be greatly reduced. Many workshops peddle unproven educational ideologies and/or technologies rather than strengthen academic content. The certificates handed out at these workshops do not necessarily measure mastery of what is taught. Wayne Bishop, a California State math professor, has gone so far as to state that "teacher workshops are the mechanism which spread educational viruses".

The PR-DOE program is currently participating in the U.S. federal government "class size reduction" program. A serious attempt should be made to increase the quality, and not just the number, of recruits to the teaching profession. Otherwise, this program is unlikely to improve the quality of education. It is better to have a classroom of 40 students with a highly competent teacher than four classrooms of 10 students each with an incompetent one. A good educational video is more effective than an incompetent teacher; the former can do less harm. Accordingly, we recommend any class size reductions be accompanied by a corresponding increase in teacher quality.

8.4 Recommendations Concerning Textbooks

The Mathematics Standards Panel should have the final say in textbook recommendations. The main criterion in selection should be conformity with the new Mathematics Standards. The list of texts for adoption should be published by the Panel with six months notice and the general public be given opportunity to comment in writing to the Panel. As previously pointed out in section 3.5, authors of school textbooks and other persons with conflicts of interest should be excluded from the review process. A report giving reasons for the selection of a given text should be available in the public domain.

Since most high school mathematics textbooks available in Puerto Rico and the U.S. are of quite low quality and in general cannot be relied upon for accuracy or completeness, it is recommended that copies of books such as the Japanese Mathematics Series for Grades 10 and 11 [41], [42], [43], and [44] be made available to all mathematics teachers, including candidate teachers, and be placed in university and school libraries. These texts provide a mathematically correct reference and serve as a guide to a minimal set of concepts and theorems that teachers and students should know and be able to prove. In the longer term, the Standards Panel should compile a list of the best available reference texts.

8.5 Recommendations Concerning Technology

We recommend that calculators and computers **not** be used in grades K-4 for the teaching of mathematics. Calculations should first be mastered mentally, then by pencil and paper, and only then by calculator. Traditional technologies, such as pencil and paper, rulers, compasses, and protractors, are still important and should not be de-emphasized. Manipulatives such as straws, rice and glue, mentioned in the current standards, are inadequate substitutes.

8.6 Concluding Remarks

Our analysis and recommendations have focused on Mathematics education. We realize that to improve our public school system a comprehensive reform effort is required. Apart from curricula and curricula related concerns, the following are among the many other factors that have to be considered: parental and community involvement, school infrastructure, counselling and disciplinary issues, administrative considerations, and reform of the Schools of Education. We hope that our article contributes to the ongoing efforts to achieve a better education system for all.

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