THE UNIVERSITY OF HULL

Final Examination: Part II

For the Special Degree of Bachelor of Science
PHYSICS V
1971

Theoretical Option
Wednesday 2nd June, 2 p.m. to 5 p.m.

SECTION A

Answer QUESTION 1 *and any other* THREE, *questions.*

1. The requirement that any general description of a physical system shall be independent of the reference frame can be fulfilled by an appeal to variational principles or to the theory of coordinate transformations. Discuss the way in which this invariance requirement is satisfied in different aspects of physics by giving one example, of your choice, from each of the four theories: (i) classical mechanics; (ii) quantum mechanics; (iii) special relativity; and (iv) electro-dynamics.

2. Show how approximate eigenvalues and eigenfunctions for an Hermitian linear operator can be found by setting up the matrix of the operator in an orthonormal basis. By means of a 2×2 matrix example, show how this approach is related to traditional perturbation theory. State how the matrix approach is modified if the basis functions are not orthonormal. Show that the Heitler-London valence bond wavefunction for H_2 is equivalent to a wavefunction involving configuration interaction from the molecular orbital viewpoint.

3. Show that Hamilton's principle leads to the Lagrange equations of motion of classical mechanics, and that the modified Hamilton's principle leads to the Hamilton canonical equations of motion. Show formally that the Lagrangian L is not unique, and give a simple example to illustrate this. Explain how this non-uniqueness of L, can be used to achieve canonical transformations in the Hamilton approach to classical mechanics.

4. Give the definition in special relativity of 4-velocity, 4-momentum and 4-force, discussing in detail the significance of these definitions from the transformation theoretic viewpoint. Hence deduce the necessary and sufficient conditions under which the proper mass (or rest mass) remains constant. Discuss briefly the physicist's concept of the nature of forces and hence give a mathematical formulation which leads, under suitable approximations, to the inverse square law of attraction between two electrically neutral material particles. Indicate briefly the range of physical circumstances under which the approximations have practical validity.

5. Explain what is meant by a gauge transformation in electromagnetic theory. Derive an expression for the vector potential $\mathbf{A}(\mathbf{r}, t)$ associated with a time-harmonic current density $\mathbf{J}(\mathbf{r})\exp(i\omega t)$ and show that under certain conditions it can be described in terms of a dipole moment $\mathbf{p}$. Hence show that for $\|\mathbf{r}\| \ll \lambda$, the electric field at any time is identical with that of an electrostatic dipole.

6. Describe in detail the formulation of an action principle for the classical electrodynamics of a system of charges and fields. Hence obtain the relativistic equations of motion of the charged particles in an electromagnetic field, and show that Maxwell's two source-free equations are satisfied identically.

7. Explain how the Boltzmann equation for the particle velocity distribution is used as a basis for the kinetic theory of a dilute gas not in equilibrium. State clearly any assumptions that are associated with the equation, stressing particularly their physical interpretation. Then
   Either–
   (a) By defining the H-function of Boltzmann, indicate how the Boltzmann equation can describe the irreversible change of the dilute gas towards an equilibrium state. Show further how the equation can lead to the recognition of the equilibrium velocity distribution (Maxwell distribution).

   Or
   (b) Explore the relationship between the Boltzmann equation and the conservation equations of mass and momentum for the macroscopic fluid flow, indicating briefly (without full derivation) how the Euler and Navier-Stokes equations arise as the result of a procedure of successive approximations.

8. Express the statements of conservation of mass, momentum and energy in a form suitable for the description of the thermal flow of an incompressible simple fluid. Show how these expressions can be converted into a form suitable for the explicit calculation of the velocity and temperature distributions within the fluid by invoking the empirical laws of Stokes and Fourier.

   Set down the boundary conditions that are to be associated with the equations you have derived. Explain in qualitative form how these boundary conditions can lead in suitable cases to the concept of the boundary layer. As an illustration of your arguments for a particular case, describe the conditions within the steady laminar boundary layer for the thermal flow of a viscous fluid in the vicinity of a semi-infinite flat plate held at zero incidence to the incoming stream.

9. Compare and contrast the three traditional ensembles of statistical mechanics and,

   Either–
   (a) Show how any one of them leads to the specification of the thermodynamic state of a classical ideal gas. Indicate briefly the steps you would take to calculate the state of a system of particles interacting through a central pair potential.

   Or–
   Show under what circumstances the canonical ensemble is equivalent to the microcanonical ensemble.